

Tri-bimaximal Mixing and Cabibbo Angle in S_4 Flavor Model with SUSY

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Abstract

We present a flavor model of quarks and leptons with the non-Abelian discrete symmetry S_4 in the framework of the $SU(5)$ SUSY GUT. Three generations of $\bar{\mathbf{5}}$ -plets in $SU(5)$ are assigned to $\mathbf{3}$ of S_4 while the first and second generations of 10-plets in $SU(5)$ are assigned to $\mathbf{2}$ of S_4 , and the third generation of 10-plet is assigned to $\mathbf{1}$ of S_4 . Right-handed neutrinos are also assigned to $\mathbf{2}$ for the first and second generations and $\mathbf{1}'$ for the third generation. We predict the Cabibbo angle as well as the tri-bimaximal mixing of neutrino flavors. We also predict the non-vanishing U_{e3} of the neutrino flavor mixing due to higher dimensional mass operators. Our predicted CKM mixing angles and the CP violation are consistent with experimental values. We also study SUSY breaking terms in the slepton sector. Our model leads to smaller values of flavor changing neutral currents than the present experimental bounds.

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1 Introduction

There are many free parameters in the standard model including its extension with neutrino mass terms and most of them are originated from the flavor sector, i.e. Yukawa couplings of quarks and leptons. Quark masses and mixing angles have been discussed in the standpoint of the flavor symmetries. The discovery of neutrino masses and the neutrino flavor mixing has stimulated the work of the flavor symmetries. Recent experiments of the neutrino oscillation go into a new phase of precise determination of mixing angles and mass squared differences [1, 2, 3, 4], which indicate the tri-bimaximal mixing for three flavors in the lepton sector [5, 6, 7, 8]. These large mixing angles are completely different from the quark mixing ones. Therefore, it is very important to find a natural model that leads to these mixing patterns of quarks and leptons with good accuracy.

The flavor symmetry is expected to explain the mass spectrum and the mixing matrix of both quarks and leptons. Especially, the non-Abelian discrete symmetry [9] has been studied intensively in the quark and lepton sectors. Actually, the tri-bimaximal mixing of leptons has been at first understood based on the non-Abelian finite group A_4 [10, 11, 12, 13, 14]. Until now, much progress has been made in the theoretical and phenomenological analysis of A_4 flavor model [15]-[73]. On the other hand, much attention has been devoted to the question whether these models can be extended to describe the observed pattern of quark masses and mixing angles, and whether these can be made compatible with the $SU(5)$ or $SO(10)$ grand unified theory (GUT). The attractive candidate is the S_4 symmetry, which has been already used for the neutrino masses and the neutrino flavor mixing [74, 75, 76, 77]. The exact tri-bimaximal neutrino mixing is realized in S_4 flavor models [78, 79, 80, 81, 82, 83, 84]. Many detail studies in the S_4 flavor model have been presented for the quark and lepton sectors [85]-[95]. There are attempts to unify the quark and lepton sectors toward a grand unified theory of flavor [86, 87, 88], however, quark mixing angles were not predicted clearly.

Recently, S_4 flavor models to unify quarks and leptons have been presented in the framework of the $SU(5)$ SUSY GUT [80] or $SO(10)$ SUSY GUT [96]. However, quantitative analyses have not been given there since the contribution from higher dimensional mass operators are not discussed. There also appeared the S_4 flavor model in $SU(5)$ SUSY GUT [97] and the Pati-Salam SUSY GUT [98], taking account of higher dimensional mass operators.

In this paper, we present another S_4 flavor model with Z_4 taking account of higher dimensional mass operators. We predict the deviation from the tri-bimaximal mixing of the lepton flavor numerically. The CKM mixing angles and CP violation are discussed numerically owing to higher dimensional mass operators. We also discuss the flavor changing neutral current (FCNC) in the SUSY sector, which is important to constrain the parameter of the flavor model.

The S_4 group has 24 distinct elements and irreducible representations **1**, **1'**, **2**, **3**, and **3'**. Three generations of $\bar{5}$ -plets in $SU(5)$ are assigned to **3** of S_4 while the first and second generations of 10-plets in $SU(5)$ are assigned to **2** of S_4 , and the third generation of 10-plet is assigned to **1** of S_4 . These assignments of S_4 for $\bar{5}$ and 10 lead to the completely different structure of quark and lepton mass matrices. Right-handed neutrinos, which are $SU(5)$ gauge singlets, are also assigned to **2** for the first and second generations, and **1'** for the third generation. These assignments realize the tri-bimaximal mixing of neutrino flavors. Gauge

singlet scalars, which are so called flavon, are introduced. in our model. Relevant vacuum alignment of flavons gives the quark flavor mixing angles as well as the tri-bimaximal mixing of neutrino flavors. Especially, the Cabibbo angle is predicted to be around 15° .

In section 2, we present the $S_4 \times Z_4 \times U(1)_{FN}$ flavor model of quarks and leptons in $SU(5)$ SUSY GUT, and discuss the effect of the higher dimensional mass operators. The deviation from the tri-bimaximal mixing is predicted, and CKM mixing angles and the CP violation are discussed in detail. In section 3, the alignment of the vacuum expectation values (VEVs) is derived. In section 4, the FCNC in the slepton sector is discussed. Section 5 is devoted to the summary. We present the multiplication rule of S_4 , the determination of $U(1)_{FN}$ quantum numbers, and the analysis of the scalar potential in Appendices A, B, and C, respectively.

2 $S_4 \times Z_4 \times U(1)_{FN}$ flavor model with $SU(5)$ SUSY GUT

2.1 Assignments of superfields

We present the S_4 flavor model in the framework of $SU(5)$ SUSY GUT. The flavor symmetry of quarks and leptons is the discrete group S_4 in our model. The group S_4 has irreducible representations **1**, **1'**, **2**, **3**, and **3'**. The multiplication rule is shown in Appendix A.

	(T_1, T_2)	T_3	(F_1, F_2, F_3)	(N_e^c, N_μ^c)	N_τ^c	H_5	$H_{\bar{5}}$	H_{45}	Θ
$SU(5)$	10	10	5	1	1	5	5	45	1
S_4	2	1	3	2	1'	1	1	1	1
Z_4	$-i$	-1	i	1	1	1	1	-1	1
$U(1)_{FN}$	ℓ	0	0	m	0	0	0	0	-1

	(χ_1, χ_2)	(χ_3, χ_4)	(χ_5, χ_6, χ_7)	$(\chi_8, \chi_9, \chi_{10})$	$(\chi_{11}, \chi_{12}, \chi_{13})$	χ_{14}
$SU(5)$	1	1	1	1	1	1
S_4	2	2	3'	3	3	1
Z_4	$-i$	1	$-i$	-1	i	i
$U(1)_{FN}$	$-\ell$	$-n$	0	0	0	$-\ell$

Table 1: Assignments of $SU(5)$, S_4 , Z_4 , and $U(1)_{FN}$ representations.

Let us present the model of the quark and lepton flavor with $SU(5)$ SUSY GUT. In $SU(5)$, matter fields are unified into 10 and $\bar{5}$ dimensional representations. Three generations of $\bar{5}$, which are denoted by F_i ($i = 1, 2, 3$), are assigned to **3** of S_4 . On the other hand, the third generation of the 10-dimensional representation is assigned to **1** of S_4 , so that the top quark Yukawa coupling is allowed in the tree level. While, the first and second generations are assigned to **2** of S_4 . These 10-dimensional representations are denoted by T_3 and (T_1, T_2) , respectively. Right-handed neutrinos, which are $SU(5)$ gauge singlets, are also assigned to **1'** and **2** for N_τ^c and (N_e^c, N_μ^c) , respectively ¹.

We introduce new scalars χ_i in addition to the 5-dimensional, $\bar{5}$ -dimensional, and 45-dimensional Higgs of $SU(5)$, H_5 , $H_{\bar{5}}$, and H_{45} , which are assigned to **1** of S_4 . These new

¹Our S_4 assignments of matter fields are same as ones in the model [97] except that right-handed neutrinos are assigned to **3** there.

scalars are supposed to be $SU(5)$ gauge singlets. Scalars (χ_1, χ_2) and (χ_3, χ_4) are assigned to **2**, (χ_5, χ_6, χ_7) are assigned to **3'**, $(\chi_8, \chi_9, \chi_{10})$ and $(\chi_{11}, \chi_{12}, \chi_{13})$ are assigned to **3**, and χ_{14} is assigned to **1** of S_4 representations, respectively. In the leading order, (χ_3, χ_4) are coupled with the right-handed Majorana neutrino sector, (χ_5, χ_6, χ_7) are coupled with the Dirac neutrino sector, $(\chi_8, \chi_9, \chi_{10})$ and $(\chi_{11}, \chi_{12}, \chi_{13})$ are coupled with the charged lepton and down-type quark sectors, respectively. In the next-to-leading order, (χ_1, χ_2) scalars are coupled with the up-type quark sector, and χ_{14} contributes to the charged lepton and down-type quark sectors, and then the mass ratio of the electron and down quark is reproduced properly. We also add Z_4 symmetry to obtain relevant couplings. In order to get the natural hierarchy among quark and lepton masses, the Froggatt-Nielsen mechanism [99] is introduced as an additional $U(1)_{FN}$ flavor symmetry, where Θ denotes the Froggatt-Nielsen flavon. The particle assignments of $SU(5)$, S_4 , Z_4 , and $U(1)_{FN}$ are summarized in Table 1. The $U(1)_{FN}$ charges ℓ , m , and n will be determined phenomenologically.

We can now write down the superpotential respecting S_4 , Z_4 , and $U(1)_{FN}$ symmetries in terms of the S_4 cutoff scale Λ , and the $U(1)_{FN}$ cutoff scale $\bar{\Lambda}$. The $SU(5)$ invariant superpotential of the Yukawa sector up to the linear terms of χ_i ($i = 1, \dots, 13$) is given as

$$\begin{aligned}
w = & y_1^u(T_1, T_2) \otimes T_3 \otimes (\chi_1, \chi_2) \otimes H_5/\Lambda + y_2^u T_3 \otimes T_3 \otimes H_5 \\
& + y_1^N(N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) \otimes \Theta^{2m}/\bar{\Lambda}^{2m-1} \\
& + y_2^N(N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) \otimes (\chi_3, \chi_4) \otimes \Theta^{2m-n}/\bar{\Lambda}^{2m-n} + M N_\tau^c \otimes N_\tau^c \\
& + y_1^D(N_e^c, N_\mu^c) \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_5 \otimes \Theta^m/(\Lambda \bar{\Lambda}^m) \\
& + y_2^D N_\tau^c \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_5/\Lambda \\
& + y_1(F_1, F_2, F_3) \otimes (T_1, T_2) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes H_{45} \otimes \Theta^\ell/(\Lambda \bar{\Lambda}^\ell) \\
& + y_2(F_1, F_2, F_3) \otimes T_3 \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_5/\Lambda,
\end{aligned} \tag{1}$$

where y_1^u , y_2^u , y_1^N , y_2^N , y_1^D , y_2^D , y_1 , and y_2 are Yukawa couplings. The $U(1)_{FN}$ charges ℓ , m , and n are integers, and satisfy the conditions $m - n < 0$ and $2m - n \geq 0$. In our numerical study, we fix $\ell = m = 1$ and $n = 2$ phenomenologically as seen in Appendix B. Then, some couplings are forbidden in the superpotential. We discuss the feature of the quark and lepton mass matrices and flavor mixing based on this superpotential. However, we will take into account the next-to-leading couplings as to χ_i in the numerical study of the flavor mixing and CP violation.

2.2 Lepton sector

We begin to discuss the lepton sector of the superpotential w . Denoting Higgs doublets as h_u and h_d , the superpotential of the Yukawa sector respecting the $S_4 \times Z_4 \times U(1)_{FN}$ symmetry is given for charged leptons as

$$\begin{aligned}
w_l = & -3y_1 \left[\frac{e^c}{\sqrt{2}}(l_\mu \chi_9 - l_\tau \chi_{10}) + \frac{\mu^c}{\sqrt{6}}(-2l_e \chi_8 + l_\mu \chi_9 + l_\tau \chi_{10}) \right] h_{45} \Theta^\ell/(\Lambda \bar{\Lambda}^\ell) \\
& + y_2 \tau^c (l_e \chi_{11} + l_\mu \chi_{12} + l_\tau \chi_{13}) h_d/\Lambda.
\end{aligned} \tag{2}$$

For right-handed Majorana neutrinos, the superpotential is given as

$$w_N = y_1^N (N_e^c N_e^c + N_\mu^c N_\mu^c) \Theta^{2m} / \bar{\Lambda}^{2m-1} + y_2^N [(N_e^c N_\mu^c + N_\mu^c N_e^c) \chi_3 + (N_e^c N_e^c - N_\mu^c N_\mu^c) \chi_4] \Theta^{2m-n} / \bar{\Lambda}^{2m-n} + M N_\tau^c N_\tau^c, \quad (3)$$

and for Dirac neutrino Yukawa couplings, the superpotential is

$$w_D = y_1^D \left[\frac{N_e^c}{\sqrt{6}} (2l_e \chi_5 - l_\mu \chi_6 - l_\tau \chi_7) + \frac{N_\mu^c}{\sqrt{2}} (l_\mu \chi_6 - l_\tau \chi_7) \right] h_u \Theta^m / (\Lambda \bar{\Lambda}^m) + y_2^D N_\tau^c (l_e \chi_5 + l_\mu \chi_6 + l_\tau \chi_7) h_u / \Lambda. \quad (4)$$

Higgs doublets h_u , h_d and gauge singlet scalars Θ , χ_i are assumed to develop their VEVs as follows:

$$\begin{aligned} \langle h_u \rangle &= v_u, & \langle h_d \rangle &= v_d, & \langle h_{45} \rangle &= v_{45}, & \langle \Theta \rangle &= \theta, \\ \langle (\chi_3, \chi_4) \rangle &= (u_3, u_4), & \langle (\chi_5, \chi_6, \chi_7) \rangle &= (u_5, u_6, u_7), \\ \langle (\chi_8, \chi_9, \chi_{10}) \rangle &= (u_8, u_9, u_{10}), & \langle (\chi_{11}, \chi_{12}, \chi_{13}) \rangle &= (u_{11}, u_{12}, u_{13}), \end{aligned} \quad (5)$$

which are supposed to be real. Then, we obtain the mass matrix for charged leptons as

$$M_l = -3y_1 \lambda^\ell v_{45} \begin{pmatrix} 0 & \alpha_9/\sqrt{2} & -\alpha_{10}/\sqrt{2} \\ -2\alpha_8/\sqrt{6} & \alpha_9/\sqrt{6} & \alpha_{10}/\sqrt{6} \\ 0 & 0 & 0 \end{pmatrix} + y_2 v_d \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_{11} & \alpha_{12} & \alpha_{13} \end{pmatrix}, \quad (6)$$

while the right-handed Majorana neutrino mass matrix is given as

$$M_N = \begin{pmatrix} \lambda^{2m-n} (y_1^N \lambda^n \bar{\Lambda} + y_2^N \alpha_4 \Lambda) & y_2^N \lambda^{2m-n} \alpha_3 \Lambda & 0 \\ y_2^N \lambda^{2m-n} \alpha_3 \Lambda & \lambda^{2m-n} (y_1^N \lambda^n \bar{\Lambda} - y_2^N \alpha_4 \Lambda) & 0 \\ 0 & 0 & M \end{pmatrix}. \quad (7)$$

Because of the condition $m - n < 0$, (1, 3), (2, 3), (3, 1), and (3, 3) elements of the right-handed Majorana neutrino mass matrix vanish. These are so called SUSY zeros. The Dirac mass matrix of neutrinos is

$$M_D = y_1^D \lambda^m v_u \begin{pmatrix} 2\alpha_5/\sqrt{6} & -\alpha_6/\sqrt{6} & -\alpha_7/\sqrt{6} \\ 0 & \alpha_6/\sqrt{2} & -\alpha_7/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} + y_2^D v_u \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_5 & \alpha_6 & \alpha_7 \end{pmatrix}, \quad (8)$$

where we denote $\alpha_i \equiv u_i/\Lambda$ and $\lambda \equiv \theta/\bar{\Lambda}$.

In order to get the left-handed mixing of charged leptons, we investigate $M_l^\dagger M_l$. If we can take vacuum alignment $(u_8, u_9, u_{10}) = (0, u_9, 0)$ and $(u_{11}, u_{12}, u_{13}) = (0, 0, u_{13})$, that is $\alpha_8 = \alpha_{10} = \alpha_{11} = \alpha_{12} = 0$, we obtain

$$M_l = \begin{pmatrix} 0 & -3y_1 \lambda^\ell \alpha_9 v_{45}/\sqrt{2} & 0 \\ 0 & -3y_1 \lambda^\ell \alpha_9 v_{45}/\sqrt{6} & 0 \\ 0 & 0 & y_2 \alpha_{13} v_d \end{pmatrix}, \quad (9)$$

then $M_l^\dagger M_l$ is as follows:

$$M_l^\dagger M_l = v_d^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 6|\bar{y}_1 \lambda^\ell \alpha_9|^2 & 0 \\ 0 & 0 & |y_2|^2 \alpha_{13}^2 \end{pmatrix}, \quad (10)$$

where we replace $y_1 v_{45}$ with $\bar{y}_1 v_d$. We find $\theta_{12}^l = \theta_{13}^l = \theta_{23}^l = 0$, where θ_{ij}^l denote left-handed mixing angles to diagonalize the charged lepton mass matrix. Then, charged lepton masses are

$$m_e^2 = 0, \quad m_\mu^2 = 6|\bar{y}_1 \lambda^\ell \alpha_9|^2 v_d^2, \quad m_\tau^2 = |y_2|^2 \alpha_{13}^2 v_d^2. \quad (11)$$

It is remarkable that the electron mass vanishes. We will discuss the electron mass as well as the down quark mass in the next-to-leading order.

Taking vacuum alignment $(u_3, u_4) = (0, u_4)$ and $(u_5, u_6, u_7) = (u_5, u_5, u_5)$ in Eqs. (7) and (8), the right-handed Majorana mass matrix of neutrinos turns to

$$M_N = \begin{pmatrix} \lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} + y_2^N \alpha_4 \Lambda) & 0 & 0 \\ 0 & \lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} - y_2^N \alpha_4 \Lambda) & 0 \\ 0 & 0 & M \end{pmatrix}, \quad (12)$$

and the Dirac mass matrix of neutrinos becomes

$$M_D = y_1^D \lambda^m v_u \begin{pmatrix} 2\alpha_5/\sqrt{6} & -\alpha_5/\sqrt{6} & -\alpha_5/\sqrt{6} \\ 0 & \alpha_5/\sqrt{2} & -\alpha_5/\sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} + y_2^D v_u \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_5 & \alpha_5 & \alpha_5 \end{pmatrix}. \quad (13)$$

By using the seesaw mechanism $M_\nu = M_D^T M_N^{-1} M_D$, the left-handed Majorana neutrino mass matrix is written as

$$M_\nu = \begin{pmatrix} a + \frac{2}{3}b & a - \frac{1}{3}b & a - \frac{1}{3}b \\ a - \frac{1}{3}b & a + \frac{1}{6}b + \frac{1}{2}c & a + \frac{1}{6}b - \frac{1}{2}c \\ a - \frac{1}{3}b & a + \frac{1}{6}b - \frac{1}{2}c & a + \frac{1}{6}b + \frac{1}{2}c \end{pmatrix}, \quad (14)$$

where

$$a = \frac{(y_2^D \alpha_5 v_u)^2}{M}, \quad b = \frac{(y_1^D \alpha_5 v_u \lambda^m)^2}{\lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} + y_2^N \alpha_4 \Lambda)}, \quad c = \frac{(y_1^D \alpha_5 v_u \lambda^m)^2}{\lambda^{2m-n}(y_1^N \lambda^n \bar{\Lambda} - y_2^N \alpha_4 \Lambda)}. \quad (15)$$

The neutrino mass matrix is decomposed as

$$M_\nu = \frac{b+c}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \frac{3a-b}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{b-c}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (16)$$

which gives the tri-bimaximal mixing matrix $U_{\text{tri-bi}}$ and mass eigenvalues as follows:

$$U_{\text{tri-bi}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad m_1 = b, \quad m_2 = 3a, \quad m_3 = c. \quad (17)$$

The next-to-leading terms of the superpotential are important to predict the deviation from the tri-bimaximal mixing of leptons, especially, U_{e3} . The relevant superpotential in the charged lepton sector is given at the next-to-leading order as

$$\begin{aligned}
\Delta w_l = & y_{\Delta_a}(T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_1, \chi_2) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_{\bar{5}}/\Lambda^2 \\
& + y_{\Delta_b}(T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes \chi_{14} \otimes H_{\bar{5}}/\Lambda^2 \\
& + y_{\Delta_c}(T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_1, \chi_2) \otimes (\chi_5, \chi_6, \chi_7) \otimes H_{45}/\Lambda^2 \\
& + y_{\Delta_d}(T_1, T_2) \otimes (F_1, F_2, F_3) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_{14} \otimes H_{45}/\Lambda^2 \\
& + y_{\Delta_e} T_3 \otimes (F_1, F_2, F_3) \otimes (\chi_5, \chi_6, \chi_7) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes H_{\bar{5}} \otimes / \Lambda^2 \\
& + y_{\Delta_f} T_3 \otimes (F_1, F_2, F_3) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_{45} \otimes / \Lambda^2 .
\end{aligned} \tag{18}$$

By using this superpotential, we obtain the charged lepton mass matrix as follows:

$$M_l \simeq \begin{pmatrix} \epsilon_{11} & \frac{\sqrt{3}m_\mu}{2} + \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \frac{m_\mu}{2} + \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & 0 & m_\tau + \epsilon_{33} \end{pmatrix}, \tag{19}$$

where m_μ and m_τ are given in Eq. (11), and ϵ_{ij} 's are calculated by using Eq. (18) as

$$\begin{aligned}
\epsilon_{11} &= y_{\Delta_b} \alpha_5 \alpha_{14} v_d - 3 \bar{y}_{\Delta_{c_2}} \alpha_1 \alpha_5 v_d, \\
\epsilon_{12} &= -\frac{1}{2} y_{\Delta_b} \alpha_5 \alpha_{14} v_d + 3 \left[\frac{\sqrt{3}}{4} (\sqrt{3} - 1) \bar{y}_{\Delta_{c_1}} - \frac{1}{4} (\sqrt{3} + 1) \bar{y}_{\Delta_{c_2}} \right] \alpha_1 \alpha_5 v_d, \\
\epsilon_{13} &= \left[\left\{ \frac{\sqrt{3}}{4} (\sqrt{3} - 1) y_{\Delta_{a_1}} + \frac{1}{4} (\sqrt{3} + 1) y_{\Delta_{a_2}} \right\} \alpha_1 \alpha_{13} - \frac{1}{2} y_{\Delta_b} \alpha_5 \alpha_{14} \right] v_d \\
&\quad - 3 \left[\left\{ -\frac{\sqrt{3}}{4} (\sqrt{3} + 1) \bar{y}_{\Delta_{c_1}} - \frac{1}{4} (\sqrt{3} - 1) \bar{y}_{\Delta_{c_2}} \right\} \alpha_1 \alpha_5 + \frac{\sqrt{3}}{2} \bar{y}_{\Delta_d} \alpha_{13} \alpha_{14} \right] v_d, \\
\epsilon_{21} &= -3 \bar{y}_{\Delta_{c_1}} \alpha_1 \alpha_5 v_d, \\
\epsilon_{22} &= \frac{\sqrt{3}}{2} y_{\Delta_b} \alpha_5 \alpha_{14} v_d + 3 \left[\frac{1}{4} (\sqrt{3} - 1) \bar{y}_{\Delta_{c_1}} + \frac{\sqrt{3}}{4} (\sqrt{3} + 1) \bar{y}_{\Delta_{c_2}} \right] \alpha_1 \alpha_5 v_d, \\
\epsilon_{23} &= \left[\left\{ -\frac{1}{4} (\sqrt{3} - 1) y_{\Delta_{a_1}} + \frac{\sqrt{3}}{4} (\sqrt{3} + 1) y_{\Delta_{a_2}} \right\} \alpha_1 \alpha_{13} - \frac{\sqrt{3}}{2} y_{\Delta_b} \alpha_5 \alpha_{14} \right] v_d \\
&\quad - 3 \left[\left\{ \frac{1}{4} (\sqrt{3} + 1) \bar{y}_{\Delta_{c_1}} - \frac{\sqrt{3}}{4} (\sqrt{3} - 1) \bar{y}_{\Delta_{c_2}} \right\} \alpha_1 \alpha_5 - \frac{1}{2} \bar{y}_{\Delta_d} \alpha_{13} \alpha_{14} \right] v_d, \\
\epsilon_{31} &= -y_{\Delta_e} \alpha_5 \alpha_9 v_d - 3 \bar{y}_{\Delta_f} \alpha_9 \alpha_{13} v_d, \\
\epsilon_{33} &= y_{\Delta_e} \alpha_5 \alpha_9 v_d.
\end{aligned} \tag{20}$$

Magnitudes of ϵ_{ij} 's are of $\mathcal{O}(\tilde{\alpha}^2)$, where $\tilde{\alpha}$ is a linear combination of α_i 's.

Then, $M_l^\dagger M_l$ is given in terms of ϵ_{ij} , which give the non-vanishing electron mass, as

follows:

$$M_l^\dagger M_l \simeq \begin{pmatrix} |\epsilon_{11}|^2 + |\epsilon_{21}|^2 + |\epsilon_{31}|^2 & \frac{1}{2}(\sqrt{3}\epsilon_{11}^* + \epsilon_{21}^*)m_\mu & \epsilon_{31}^* m_\tau \\ \frac{1}{2}(\sqrt{3}\epsilon_{11} + \epsilon_{21})m_\mu & m_\mu^2 & \frac{1}{2}(\sqrt{3}\epsilon_{13} + \epsilon_{23})m_\mu \\ \epsilon_{31}m_\tau & \frac{1}{2}(\sqrt{3}\epsilon_{13}^* + \epsilon_{23}^*)m_\mu & m_\tau^2 \end{pmatrix}. \quad (21)$$

Thus, the charged lepton mass matrix is not diagonal due to next-to-leading terms ϵ_{ij} , which give the non-vanishing electron mass. Since we have $m_\mu = \mathcal{O}(\lambda\tilde{\alpha})$, $m_\tau = \mathcal{O}(\tilde{\alpha})$, and $\epsilon_{ij} = \mathcal{O}(m_e)$, mixing angles θ_{12}^l , θ_{13}^l and θ_{23}^l are given as

$$\theta_{12}^l = \mathcal{O}\left(\frac{m_e}{m_\mu}\right), \quad \theta_{13}^l = \mathcal{O}\left(\frac{m_e}{m_\tau}\right), \quad \theta_{23}^l = \mathcal{O}\left(\frac{m_e m_\mu}{m_\tau^2}\right). \quad (22)$$

Therefore, the charged lepton mixing matrix is written as

$$U_E = \begin{pmatrix} 1 & \mathcal{O}\left(\frac{m_e}{m_\mu}\right) & \mathcal{O}\left(\frac{m_e}{m_\tau}\right) \\ \mathcal{O}\left(\frac{m_e}{m_\mu}\right) & 1 & \mathcal{O}\left(\frac{m_e m_\mu}{m_\tau^2}\right) \\ \mathcal{O}\left(\frac{m_e}{m_\tau}\right) & \mathcal{O}\left(\frac{m_e m_\mu}{m_\tau^2}\right) & 1 \end{pmatrix}. \quad (23)$$

Now, the lepton mixing matrix U is deviated from the tri-bimaximal mixing as follows:

$$U = U_E^\dagger U_{\text{tri-bi}}. \quad (24)$$

The lepton mixing matrix elements U_{e3} , U_{e2} , and $U_{\mu 3}$ are given as

$$|U_{e3}| \sim \frac{1}{\sqrt{2}} \left(\mathcal{O}\left(\frac{m_e}{m_\mu}\right) \right), \quad |U_{e2}| \sim \frac{1}{\sqrt{3}} \left(1 + \mathcal{O}\left(\frac{m_e}{m_\mu}\right) \right), \quad |U_{\mu 3}| \sim \frac{1}{\sqrt{2}} \left(1 - \mathcal{O}\left(\frac{m_e m_\mu}{m_\tau^2}\right) \right). \quad (25)$$

Thus, the deviation from the tri-bimaximal mixing is lower than $\mathcal{O}(0.01)$, which is rather small.

Let us discuss the electron and down quark masses. The determinant of $M_l^\dagger M_l$ is

$$\det [M_l^\dagger M_l] \simeq \frac{3}{2} m_\mu^2 m_\tau^2 \left(\frac{1}{6} \epsilon_{11}^2 - \frac{1}{\sqrt{3}} \epsilon_{11} \epsilon_{21} + \frac{1}{2} \epsilon_{21}^2 \right), \quad (26)$$

where ϵ_{ij} 's are taken to be real for simplicity. Then the electron mass is given as

$$\begin{aligned} m_e^2 &\simeq \frac{3}{2} \left(\frac{1}{6} \epsilon_{11}^2 - \frac{1}{\sqrt{3}} \epsilon_{11} \epsilon_{21} + \frac{1}{2} \epsilon_{21}^2 \right) \\ &\simeq \frac{3}{2} \left[\frac{1}{6} y_{\Delta_d}^2 \alpha_5^2 \alpha_{14}^2 + y_{\Delta_d} (\sqrt{3} \bar{y}_{\Delta_{e_1}} - \bar{y}_{\Delta_{e_2}}) \alpha_1 \alpha_5^2 \alpha_{14} + \frac{1}{2} \left(3 \bar{y}_{\Delta_{e_1}} - \sqrt{3} \bar{y}_{\Delta_{e_2}} \right)^2 \alpha_1^2 \alpha_5^2 \right] v_d^2. \end{aligned} \quad (27)$$

In the same way, the down quark mass, which is discussed in subsection 2.3, is obtained as

$$m_d^2 \simeq \frac{3}{2} \left[\frac{1}{6} y_{\Delta_d}^2 \alpha_5^2 \alpha_{14}^2 - \frac{1}{3} y_{\Delta_d} (\sqrt{3} \bar{y}_{\Delta_{e_1}} - \bar{y}_{\Delta_{e_2}}) \alpha_1 \alpha_5^2 \alpha_{14} + \frac{1}{18} \left(3 \bar{y}_{\Delta_{e_1}} - \sqrt{3} \bar{y}_{\Delta_{e_2}} \right)^2 \alpha_1^2 \alpha_5^2 \right] v_d^2. \quad (28)$$

In order to get the ratio $m_e^2 : m_d^2 = 1 : 9$, we require the following condition:

$$\alpha_{14} = -\frac{5(\sqrt{3}\bar{y}_{\Delta_{e1}} - \bar{y}_{\Delta_{e2}})}{y_{\Delta_d}}\alpha_1, \quad \text{or} \quad \alpha_{14} = -\frac{2(\sqrt{3}\bar{y}_{\Delta_{e1}} - \bar{y}_{\Delta_{e2}})}{y_{\Delta_d}}\alpha_1. \quad (29)$$

Thus, the flavon χ_{14} is introduced in our model to explain the proper ratio of the electron mass and the down quark mass although those masses appear at the next-to-leading order.

Hereafter, we fix $\ell = 1$, $m = 1$, and $n = 2$ as Frogatt-Nielsen charges, which are given in Appendix B. The superpotential of the next-to-leading order for Majorana neutrinos is

$$\begin{aligned} \Delta w_N = & y_{\Delta_1}^N (N_e^c, N_\mu^c) \otimes (N_e^c, N_\mu^c) \otimes (\chi_1, \chi_2) \otimes \chi_{14}/\Lambda \\ & + y_{\Delta_2}^N (N_e^c, N_\mu^c) \otimes N_\tau^c \otimes (\chi_5, \chi_6, \chi_7) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \Theta/(\Lambda\bar{\Lambda}) \\ & + y_{\Delta_3}^N (N_e^c, N_\mu^c) \otimes N_\tau^c \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes \Theta/(\Lambda\bar{\Lambda}) \\ & + y_{\Delta_4}^N N_\tau^c \otimes N_\tau^c \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_8, \chi_9, \chi_{10})/\Lambda. \end{aligned} \quad (30)$$

The dominant matrix elements of Majorana neutrinos at the next-to-leading order are written as follows:

$$\Delta M_N = \Lambda \times \begin{pmatrix} y_{\Delta_1}^N \alpha_1 \alpha_{14} & y_{\Delta_1}^N \alpha_1 \alpha_{14} & -\frac{\lambda}{\sqrt{6}} y_{\Delta_2}^N \alpha_5 \alpha_{13} + \frac{\lambda}{\sqrt{2}} y_{\Delta_3}^N \lambda \alpha_9^2 \\ y_{\Delta_1}^N \alpha_1 \alpha_{14} & -y_{\Delta_1}^N \alpha_1 \alpha_{14} & -\frac{\lambda}{\sqrt{2}} y_{\Delta_2}^N \alpha_5 \alpha_{13} + \frac{\lambda}{\sqrt{6}} y_{\Delta_3}^N \alpha_9^2 \\ -\frac{\lambda}{\sqrt{6}} y_{\Delta_2}^N \alpha_5 \alpha_{13} + \frac{\lambda}{\sqrt{2}} y_{\Delta_3}^N \alpha_9^2 & -\frac{\lambda}{\sqrt{2}} y_{\Delta_2}^N \alpha_5 \alpha_{13} + \frac{\lambda}{\sqrt{6}} y_{\Delta_3}^N \alpha_9^2 & y_{\Delta_4}^N \alpha_9^2 \end{pmatrix}. \quad (31)$$

Then, the U_{e3} is estimated as

$$U_{e3} \sim \frac{y_{\Delta_1}^N \alpha_1 \alpha_{14}}{y_{\Delta_4}^N \alpha_9^2} \sim \mathcal{O}(\tilde{\alpha}). \quad (32)$$

We also consider the Dirac neutrino mass matrix. The superpotential at the next-to-leading order for Dirac neutrinos is given as

$$\Delta w_D = y_{\Delta}^D (N_e^c, N_\mu^c) \otimes (F_1, F_2, F_3) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes H_5 \otimes \Theta/(\Lambda^2 \bar{\Lambda}). \quad (33)$$

The dominant matrix elements of the Dirac neutrinos at the next-to-leading order are written as follows:

$$\Delta M_D = \begin{pmatrix} * & * & * \\ y_{\Delta}^D \lambda \alpha_9 \alpha_{13} v_u & * & * \\ * & * & * \end{pmatrix}. \quad (34)$$

Then, we can estimate U_{e3} as

$$U_{e3} \sim -\frac{\sqrt{6} y_{\Delta}^D \alpha_9 \alpha_{13}}{3 y_1^D \alpha_5} \sim \mathcal{O}(\tilde{\alpha}). \quad (35)$$

Thus, the contribution of the next-to-leading terms on U_{e3} is of $\mathcal{O}(\tilde{\alpha})$ in the neutrino sector while that is $\mathcal{O}(m_e/m_\mu)$ in the charged lepton sector. Therefore, it is concluded that the deviation from the tri-bimaximal mixing mainly comes from the neutrino sector.

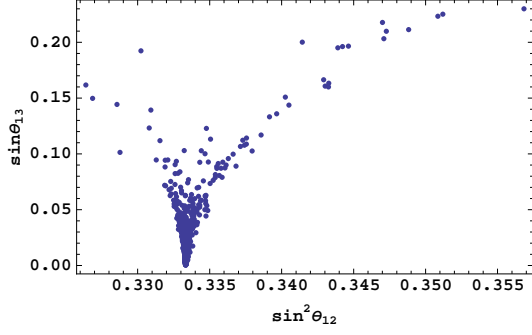


Figure 1: The allowed region on $\sin^2 \theta_{12}$ - $\sin \theta_{13}$ plane.

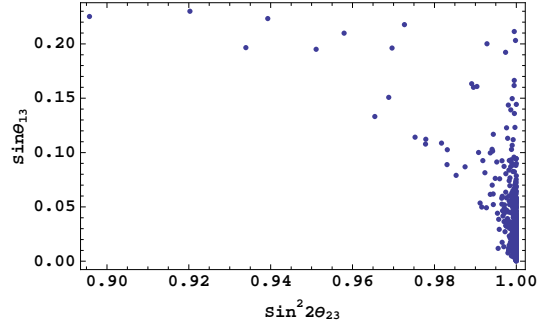


Figure 2: The allowed region on $\sin^2 2\theta_{23}$ - $\sin \theta_{13}$ plane.

Let us discuss the deviation from the tri-bimaximal mixing numerically. In order to obtain numerical result, all Yukawa couplings at the leading order are complex, and these absolute values and phases are taken to be randomly chosen from 0 to 1 and $-\pi$ to π , respectively. Other Yukawa couplings at the next-to-leading order are to be real and randomly chosen from -1 to 1 since the contribution from these phases on the CP violation is very small. Parameters α_i 's are fixed as seen in section 3. We also take $M = 10^{12}$ GeV. The parameter α_{14} is constrained to reproduce the proper ratio of the electron mass and down quark mass as seen in Eqs. (27) and (28). Yukawa couplings are also constrained to give the absolute value of the electron mass.

We present numerical result of the deviation from the tri-bimaximal mixing with scattering plots. Here, we neglect the renormalization effect of the neutrino mass matrix because we suppose the normal hierarchy of neutrino masses and take small $\tan \beta$ ($= 3$).

We show our prediction of $\sin \theta_{13}$ versus $\sin^2 \theta_{12}$ in Figure 1, where θ_{ij} 's are lepton mixing angles in the usual convention. That is, $\sin \theta_{13} = |U_{e3}|$. In Figure 2, we show the prediction of $\sin \theta_{13}$ on the $\sin^2 2\theta_{23}$ - $\sin^2 \theta_{13}$ plane. The predicted upper bound of $\sin \theta_{13}$ could be larger than 0.1 by tuning Yukawa couplings. In the case that $\sin \theta_{13}$ is larger than 0.15, the value of $\sin^2 \theta_{12}$ considerably deviates from the tri-maximum value $1/3$, that is $\sin^2 \theta_{12} \geq 0.34$. However, the predicted points of $\sin \theta_{13}$ distribute mainly in the region lower than 0.07. The mixing angle θ_{23} is mainly predicted near the maximal mixing angle $\pi/4$. The predicted points larger than 0.99 for $\sin^2 2\theta_{23}$ cover 99%.

We investigate precisely the predicted value $\sin \theta_{13}$ in our model. Let us calculate the expectation value of θ_{13} . In our calculation, one million parameter sets are generated randomly. The number of allowed parameter sets in experimental constraints is 1442. These points have been plotted in Figures 1 and 2. In Figure 3, we present the distribution of the plot versus $\sin \theta_{13}$. By using this result, we can

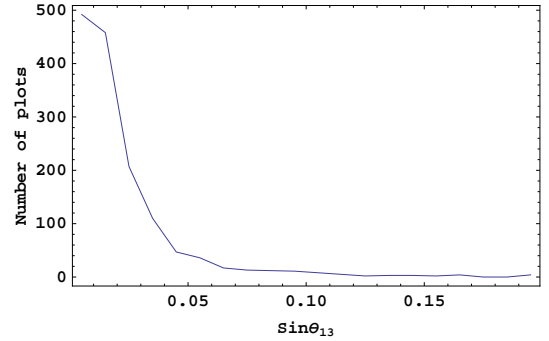


Figure 3: Number of points versus $\sin \theta_{13}$.

calculate the mean value of θ_{13} and the standard deviation. The mean value of $\sin \theta_{13}$ is 0.023 and the standard deviation is 0.028. Thus, the expected value of θ_{13} is small as expected of $\mathcal{O}(\tilde{\alpha})$ in Eqs. (32) and (35).

It is noted that the main contribution on $\sin \theta_{13}$ comes from the next-to-leading term in Majorana mass matrix ΔM_N in Eq. (31), which is a few times larger than the correction ΔM_D of Eq. (34).

We can estimate the leptonic CP violating measure J_{CP} since Yukawa couplings are taken to be complex. In Figure 4, we show the J_{CP} versus $\sin \theta_{13}$. The upper bound of J_{CP} could be larger than 0.01, which may encourage the measurement of the CP violation in the future neutrino oscillation experiments. However, the probable value of J_{CP} is much smaller than 0.01. We have estimated the mean value and its standard deviation of J_{CP} . The predicted mean value of J_{CP} is 2.1×10^{-3} and its standard deviation is 4.5×10^{-3} .

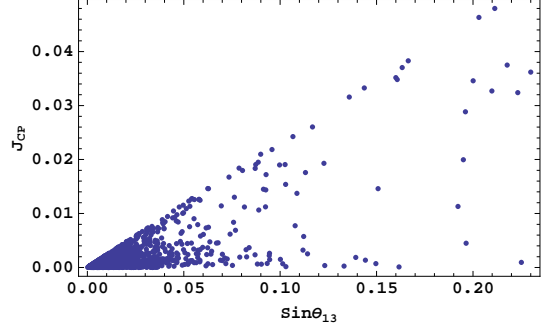


Figure 4: The allowed region on $\sin \theta_{13}$ – J_{CP} plane.

2.3 Quark sector

Let us discuss the quark sector. For down-type quarks, we can write the superpotential as follows:

$$w_d = y_1 \left[\frac{1}{\sqrt{2}}(s^c \chi_9 - b^c \chi_{10})q_1 + \frac{1}{\sqrt{6}}(-2d^c \chi_8 + s^c \chi_9 + b^c \chi_{10})q_2 \right] h_{45} \Theta^\ell / (\Lambda \bar{\Lambda}^\ell) \\ + y_2(d^c \chi_{11} + s^c \chi_{12} + b^c \chi_{13})q_3 h_d / \Lambda. \quad (36)$$

Since the vacuum alignment is fixed in the lepton sector as seen in Eq. (5), the down-type quark mass matrix at the leading order is given as

$$M_d = v_d \begin{pmatrix} 0 & 0 & 0 \\ \bar{y}_1 \lambda^\ell \alpha_9 / \sqrt{2} & \bar{y}_1 \lambda^\ell \alpha_9 / \sqrt{6} & 0 \\ 0 & 0 & y_2 \alpha_{13} \end{pmatrix}, \quad (37)$$

where we denote $\bar{y}_1 v_d = y_1 v_{45}$. Then, we have

$$M_d^\dagger M_d = v_d^2 \begin{pmatrix} \frac{1}{2} |\bar{y}_1 \lambda^\ell \alpha_9|^2 & \frac{1}{2\sqrt{3}} |\bar{y}_1 \lambda^\ell \alpha_9|^2 & 0 \\ \frac{1}{2\sqrt{3}} |\bar{y}_1 \lambda^\ell \alpha_9|^2 & \frac{1}{6} |\bar{y}_1 \lambda^\ell \alpha_9|^2 & 0 \\ 0 & 0 & |y_2|^2 \alpha_{13}^2 \end{pmatrix}. \quad (38)$$

This matrix can be diagonalized by the orthogonal matrix $U_d^{(0)}$ as

$$U_d^{(0)} = \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (39)$$

The down-type quark masses are given as

$$m_d^2 = 0, \quad m_s^2 = \frac{2}{3} |\bar{y}_1 \lambda^\ell \alpha_9|^2 v_d^2, \quad m_b^2 = |y_2|^2 \alpha_{13}^2 v_d^2, \quad (40)$$

which correspond to ones of charged lepton masses in Eq. (11). The down quark mass vanishes as well as the electron mass, however tiny masses appear at the next-to-leading order.

The down-type quark mass matrix including the next-to-leading order is

$$M_d \simeq \begin{pmatrix} \bar{\epsilon}_{11} & \bar{\epsilon}_{21} & \bar{\epsilon}_{31} \\ \frac{\sqrt{3}m_s}{2} + \bar{\epsilon}_{12} & \frac{m_s}{2} + \bar{\epsilon}_{22} & \bar{\epsilon}_{32} \\ \bar{\epsilon}_{13} & \bar{\epsilon}_{23} & m_b + \bar{\epsilon}_{33} \end{pmatrix}, \quad (41)$$

where $\bar{\epsilon}_{ij}$'s are given by replacing \bar{y}_{Δ_i} with $-\bar{y}_{\Delta_i}/3$ ($i = c_1, c_2, d, f$) in Eq. (20), and m_s and m_b are given in Eq. (40).

In order to get the left-handed mixing, we estimate $M_d^\dagger M_d$ as

$$M_d^\dagger M_d \simeq \begin{pmatrix} |\frac{\sqrt{3}m_s}{2} + \bar{\epsilon}_{12}|^2 + |\bar{\epsilon}_{11}|^2 + |\bar{\epsilon}_{13}|^2 & (\frac{\sqrt{3}}{2}m_s + \bar{\epsilon}_{12}^*)(\frac{1}{2}m_s + \bar{\epsilon}_{22}) + \bar{\epsilon}_{11}^*\bar{\epsilon}_{21} + \bar{\epsilon}_{13}^*\bar{\epsilon}_{23} & \bar{\epsilon}_{13}^*m_b \\ (\frac{\sqrt{3}}{2}m_s + \bar{\epsilon}_{12})(\frac{1}{2}m_s + \bar{\epsilon}_{22}) + \bar{\epsilon}_{11}\bar{\epsilon}_{21}^* + \bar{\epsilon}_{13}\bar{\epsilon}_{23}^* & |\frac{m_s}{2} + \bar{\epsilon}_{22}|^2 + |\bar{\epsilon}_{21}|^2 + |\bar{\epsilon}_{23}|^2 & \bar{\epsilon}_{23}^*m_b \\ \bar{\epsilon}_{13}m_b & \bar{\epsilon}_{23}m_b & m_b^2 \end{pmatrix}, \quad (42)$$

By rotating the matrix $M_d^\dagger M_d$ with the mixing matrix $U_d^{(0)}$ in Eq. (39), we have

$$U_d^{(0)\dagger} M_d^\dagger M_d U_d^{(0)} \simeq \begin{pmatrix} m_d^2 & \mathcal{O}(m_d m_s) & \frac{1}{2}(\bar{\epsilon}_{13}^* - \sqrt{3}\bar{\epsilon}_{23}^*)m_b \\ \mathcal{O}(m_d m_s) & m_s^2 & \frac{1}{2}(\sqrt{3}\bar{\epsilon}_{13}^* + \bar{\epsilon}_{23}^*)m_b \\ \frac{1}{2}(\bar{\epsilon}_{13} - \sqrt{3}\bar{\epsilon}_{23})m_b & \frac{1}{2}(\sqrt{3}\bar{\epsilon}_{13} + \bar{\epsilon}_{23})m_b & m_b^2 \end{pmatrix}. \quad (43)$$

Then, we get mixing angles θ_{12}^d , θ_{13}^d , θ_{23}^d in the mass matrix of Eq. (43) as

$$\theta_{12}^d = \mathcal{O}\left(\frac{m_d}{m_s}\right) = \mathcal{O}(0.05), \quad \theta_{13}^d = \mathcal{O}\left(\frac{m_d}{m_b}\right) = \mathcal{O}(0.005), \quad \theta_{23}^d = \mathcal{O}\left(\frac{m_d}{m_b}\right) = \mathcal{O}(0.005), \quad (44)$$

where CP violating phases are neglected.

Let us discuss the up-type quark sector. The superpotential respecting $S_4 \times Z_4 \times U(1)_{FN}$ is given as

$$w_u = y_1^u [(u^c \chi_1 + c^c \chi_2) q_3 + t^c (q_1 \chi_1 + q_2 \chi_2)] h_u / \Lambda + y_2^u t^c q_3 h_u. \quad (45)$$

We denote their VEVs as follows:

$$\langle (\chi_1, \chi_2) \rangle = (u_1, u_2). \quad (46)$$

Then, we obtain the mass matrix for up-type quarks is given as

$$M_u = v_u \begin{pmatrix} 0 & 0 & y_1^u \alpha_1 \\ 0 & 0 & y_1^u \alpha_2 \\ y_1^u \alpha_1 & y_1^u \alpha_2 & y_2^u \end{pmatrix}. \quad (47)$$

The next-to-leading terms of the superpotential are also important for the prediction of the CP violation in the quark sector. The relevant superpotential is given at the next-to-leading order as

$$\begin{aligned}\Delta w_u &= y_{\Delta_a}^u(T_1, T_2) \otimes (T_1, T_2) \otimes (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes H_5/\Lambda^2 \\ &+ y_{\Delta_b}^u(T_1, T_2) \otimes (T_1, T_2) \otimes \chi_{14} \otimes \chi_{14} \otimes H_5/\Lambda^2 \\ &+ y_{\Delta_c}^u T_3 \otimes T_3 \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes H_5/\Lambda^2.\end{aligned}\quad (48)$$

Then, the next-to-leading mass matrix becomes

$$\Delta M_u = v_u \times \begin{pmatrix} y_{\Delta_{a_1}}^u(\alpha_1^2 + \alpha_2^2) + y_{\Delta_{a_2}}^u(\alpha_1^2 - \alpha_2^2) + y_{\Delta_b}^u \alpha_{14}^2 & y_{\Delta_{a_2}}^u \alpha_1 \alpha_2 & 0 \\ y_{\Delta_{a_2}}^u \alpha_1 \alpha_2 & y_{\Delta_{a_1}}^u(\alpha_1^2 + \alpha_2^2) - y_{\Delta_{a_2}}^u(\alpha_1^2 - \alpha_2^2) + y_{\Delta_b}^u \alpha_{14}^2 & 0 \\ 0 & 0 & y_{\Delta_c}^u \alpha_9^2 \end{pmatrix}, \quad (49)$$

which is added to the leading up-type quark one of Eq. (47). In order to get the realistic quark mixing, we take the alignment

$$\alpha_1 = \alpha_2, \quad (50)$$

then, the mass matrix of the up-type quarks is

$$M_u = v_u \begin{pmatrix} 2y_{\Delta_{a_1}}^u \alpha_1^2 + y_{\Delta_b}^u \alpha_{14}^2 & y_{\Delta_{a_2}}^u \alpha_1^2 & y_1^u \alpha_1 \\ y_{\Delta_{a_2}}^u \alpha_1^2 & 2y_{\Delta_{a_1}}^u \alpha_1^2 + y_{\Delta_b}^u \alpha_{14}^2 & y_1^u \alpha_1 \\ y_1^u \alpha_1 & y_1^u \alpha_1 & y_2^u + y_{\Delta_c}^u \alpha_9^2 \end{pmatrix}. \quad (51)$$

After rotating M_u by the orthogonal matrix $U_u^{(0)}$ as

$$U_u^{(0)} = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (52)$$

we get

$$\hat{M}_u = U_u^\dagger M_u U_u = v_u \begin{pmatrix} (2y_{\Delta_{a_1}}^u - y_{\Delta_{a_2}}^u) \alpha_1^2 + y_{\Delta_b}^u \alpha_{14}^2 & 0 & 0 \\ 0 & (2y_{\Delta_{a_1}}^u + y_{\Delta_{a_2}}^u) \alpha_1^2 + y_{\Delta_b}^u \alpha_{14}^2 & \sqrt{2} y_1^u \alpha_1 \\ 0 & \sqrt{2} y_1^u \alpha_1 & y_2^u + y_{\Delta_c}^u \alpha_9^2 \end{pmatrix}. \quad (53)$$

We take a phase convention in which $(1, 1)$ and $(3, 3)$ elements in Eq. (53) are real. It is found that the magnitude of $(2, 2)$ element is much smaller than that of $(2, 3)$ and $(3, 3)$ elements. In the limit of neglecting the $(2, 2)$ element, the mass matrix \hat{M}_u is taken to be real since other phases can be removed by the phase matrix P

$$P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\rho} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (54)$$

The matrix is diagonalized by the orthogonal transformation as $V_u^T \hat{M}_u V_u$, where

$$V_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_t & r_c \\ 0 & -r_c & r_t \end{pmatrix}, \quad r_c = \sqrt{\frac{m_c}{m_c + m_t}}, \quad r_t = \sqrt{\frac{m_t}{m_c + m_t}}, \quad (55)$$

in which mass eigenvalues of up-type quarks are given as

$$m_u = \left[(2y_{\Delta_{a_1}}^u - y_{\Delta_{a_2}}^u) \alpha_1^2 + y_{\Delta_b}^u \alpha_{14}^2 \right] v_u, \\ m_c \simeq \frac{y_2^u \left[(2y_{\Delta_{a_1}}^u + y_{\Delta_{a_2}}^u) \alpha_1^2 + y_{\Delta_b}^u \alpha_{14}^2 \right] - 2y_1^{u2} \alpha_1^2}{y_2^u} v_u, \quad m_t \simeq y_2^u v_u. \quad (56)$$

Now we can discuss the CKM matrix. Mixing matrices of up- and down-type quarks are summarized as

$$U_u \simeq U_u^{(0)} P V_u = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ & 0 \\ -\sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\rho} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_t & r_c \\ 0 & -r_c & r_t \end{pmatrix}, \\ U_d \simeq \begin{pmatrix} \cos 60^\circ & \sin 60^\circ & 0 \\ -\sin 60^\circ & \cos 60^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta_{12}^d & \theta_{13}^d \\ -\theta_{12}^d - \theta_{13}^d \theta_{23}^d & 1 & \theta_{23}^d \\ -\theta_{13}^d + \theta_{12}^d \theta_{23}^d & -\theta_{23}^d - \theta_{12}^d \theta_{13}^d & 1 \end{pmatrix}. \quad (57)$$

Therefore, the CKM matrix at the GUT scale can be written as

$$V^0 = U_u^\dagger U_d \simeq \begin{pmatrix} 1 & 0 & 0 \\ 0 & r_t & -r_c \\ 0 & r_c & r_t \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\rho} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \times \begin{pmatrix} \cos 15^\circ & \sin 15^\circ & 0 \\ -\sin 15^\circ & \cos 15^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta_{12}^d & \theta_{13}^d \\ -\theta_{12}^d - \theta_{13}^d \theta_{23}^d & 1 & \theta_{23}^d \\ -\theta_{13}^d + \theta_{12}^d \theta_{23}^d & -\theta_{23}^d - \theta_{12}^d \theta_{13}^d & 1 \end{pmatrix}. \quad (58)$$

In the following, we suppose that θ_{12}^d , θ_{13}^d , and θ_{23}^d are real. Then, nine CKM matrix elements at the GUT scale are expressed as

$$V_{ud}^0 \simeq \cos 15^\circ - (\theta_{12}^d + \theta_{13}^d \theta_{23}^d) \sin 15^\circ, \\ V_{us}^0 \simeq \theta_{12}^d \cos 15^\circ + \sin 15^\circ, \\ V_{ub}^0 \simeq \theta_{13}^d \cos 15^\circ + \theta_{23}^d \sin 15^\circ, \\ V_{cd}^0 \simeq -r_t e^{i\rho} \sin 15^\circ - r_t (\theta_{12}^d + \theta_{13}^d \theta_{23}^d) e^{i\rho} \cos 15^\circ + r_c (\theta_{13}^d - \theta_{12}^d \theta_{23}^d), \\ V_{cs}^0 \simeq -r_t \theta_{12}^d e^{i\rho} \sin 15^\circ + r_t e^{i\rho} \cos 15^\circ + r_c (\theta_{23}^d + \theta_{12}^d \theta_{13}^d), \\ V_{cb}^0 \simeq -r_t \theta_{13}^d e^{i\rho} \sin 15^\circ + r_t \theta_{23}^d e^{i\rho} \cos 15^\circ - r_c, \\ V_{td}^0 \simeq -r_c \sin 15^\circ e^{i\rho} - r_c (\theta_{12}^d + \theta_{13}^d \theta_{23}^d) e^{i\rho} \cos 15^\circ + r_t (-\theta_{13}^d + \theta_{12}^d \theta_{23}^d), \\ V_{ts}^0 \simeq -r_c \theta_{12}^d \sin 15^\circ e^{i\rho} + r_c e^{i\rho} \cos 15^\circ - r_t (\theta_{23}^d + \theta_{12}^d \theta_{13}^d), \\ V_{tb}^0 \simeq -r_c \theta_{13}^d \sin 15^\circ e^{i\rho} + r_c \theta_{23}^d e^{i\rho} \cos 15^\circ + r_t. \quad (59)$$

Putting typical masses at the GUT scale $m_u = 1.04 \times 10^{-3}$ GeV, $m_c = 302 \times 10^{-3}$ GeV, and $m_t = 129$ GeV [100], we can write CKM matrix elements in terms of ρ and θ_{ij}^d at the GUT scale. We should take account the renormalization effect in order to get the CKM matrix elements at the electroweak (EW) scale. We use a simple formula of the renormalization for the CKM matrix in Ref. [101]. Then, the CKM matrix at the GUT scale [101] becomes

$$\begin{pmatrix} V_{ud}^0 & V_{us}^0 & V_{ub}^0/h(t) \\ V_{cd}^0 & V_{cs}^0 & V_{cb}^0/h(t) \\ V_{td}^0/h(t) & V_{ts}^0/h(t) & V_{tb}^0 \end{pmatrix}_{\text{EW}}, \quad (60)$$

at the EW scale. In the case of the GUT scale 10^{16} GeV, we have $h(t) \simeq 1.05$. Putting central values of the observed CKM matrix elements in the particle data group [103], i.e. $|V_{us}| = 0.2257$, $|V_{ub}| = 0.00359$, $|V_{cb}| = 0.0415$, and $|V_{td}| = 0.00874$, we obtain a parameter set

$$\rho = 123^\circ, \quad \theta_{12}^d = -0.0340, \quad \theta_{13}^d = 0.00626, \quad \theta_{23}^d = -0.00880, \quad (61)$$

which reproduce the experimental data. These magnitudes of θ_{ij}^d 's are consistent with the ones in Eq. (44).

In terms of a phase ρ , we can also estimate the magnitude of the CP violation. Let us calculate the CP violation measure, Jarlskog invariant J_{CP} [102], which is given as

$$|J_{CP}| = |\text{Im} \{V_{us}V_{cs}^*V_{ub}V_{cb}^*\}| \simeq 3.06 \times 10^{-5}, \quad (62)$$

where $\rho = 123^\circ$ is taken. Our prediction is consistent with the experimental value $J_{CP} = (3.05_{-0.20}^{+0.19}) \times 10^{-5}$ [103].

Next, we calculate three angles of the unitarity triangle, α (or ϕ_2), β (or ϕ_1), and γ (or ϕ_3),

$$\alpha = \arg \left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right), \quad \beta = \arg \left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right), \quad \gamma = \arg \left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right). \quad (63)$$

Putting $\rho = 123^\circ$, we obtain $\alpha = 89.4^\circ$, $\sin 2\beta = 0.693$ ($\beta = 21.9^\circ$), and $\gamma = 68.7^\circ$, which are consistent with experimental values $\alpha = (88_{-5}^{+6})^\circ$, $\sin 2\beta = 0.681 \pm 0.025$, and $\gamma = (77_{-32}^{+30})^\circ$ [103].

3 Magnitudes of VEVs and Alignment

As seen in previous section, we need relevant vacuum alignment to get the tri-bimaximal mixing of leptons and Cabibbo angle of quarks. The alignment of VEVs is summarized as

$$\begin{aligned} (\chi_1, \chi_2) &= (1, 1), & (\chi_3, \chi_4) &= (0, 1), \\ (\chi_5, \chi_6, \chi_7) &= (1, 1, 1), & (\chi_8, \chi_9, \chi_{10}) &= (0, 1, 0), & (\chi_{11}, \chi_{12}, \chi_{13}) &= (0, 0, 1), \end{aligned} \quad (64)$$

where these magnitudes are given in arbitrary units.

Magnitudes of $\alpha_i \equiv \langle \chi_i \rangle / \Lambda$ are determined when the quark and lepton masses are put, except for α_{14} , which appears at the next-to-leading order. Here, we have fixed $\ell = 1$, $m = 1$,

and $n = 2$ as Frogatt-Nielsen charges, which are discussed in Appendix B. Then, these are given as

$$\begin{aligned}
\alpha_3 &= \alpha_8 = \alpha_{10} = \alpha_{11} = \alpha_{12} = 0, \\
\alpha_1 &= \alpha_2 \simeq \sqrt{\frac{m_c}{2 \left| y_{\Delta_{a_2}}^u - \frac{y_1^{u^2}}{y_2^u} \right|}} v_u, \\
\alpha_4 &= \frac{(y_1^D \lambda)^2 (m_3 - m_1) m_2 M}{6 y_2^N y_2^{D^2} m_1 m_3 \Lambda}, \quad \alpha_5 = \alpha_6 = \alpha_7 = \frac{\sqrt{m_2 M}}{\sqrt{3} y_2^D v_u}, \\
\alpha_9 &= \frac{m_\mu}{\sqrt{6} |\bar{y}_1| \lambda v_d}, \quad \alpha_{13} = \frac{m_\tau}{y_2 v_d}.
\end{aligned} \tag{65}$$

where masses of quarks and leptons are given at the GUT scale.

Putting typical values of quark masses at the GUT scale [100], $M = 10^{12}$ GeV, $\lambda = 0.1$, and $\tan \beta = 3$ ($v_d \simeq 55$ GeV, $v_u \simeq 165$ GeV) with of order 1 for absolute values of Yukawa couplings, we have

$$\begin{aligned}
\alpha_1 &\sim 3 \times 10^{-2}, & \alpha_4 &\sim 10^{-2}, & \alpha_5 &\sim 10^{-2}, \\
\alpha_9 &\sim 5 \times 10^{-3}, & \alpha_{13} &\sim 2 \times 10^{-2}.
\end{aligned} \tag{66}$$

Therefore, the magnitudes of all VEVs are almost of order 10^{-2} . Hereafter, we denote the averaged value as $\tilde{\alpha}$.

We can generate the vacuum alignment through F -terms by coupling flavons to driving fields, which carry the R charge $+2$ under $U(1)_R$ symmetry.

	(χ_1, χ_2)	(χ_3, χ_4)	(χ_5, χ_6, χ_7)	$(\chi_8, \chi_9, \chi_{10})$	$(\chi_{11}, \chi_{12}, \chi_{13})$	χ_{14}
$SU(5)$	1	1	1	1	1	1
S_4	2	2	3'	3	3	1
Z_4	$-i$	1	$-i$	-1	i	i
$U(1)_{FN}$	$-\ell$	$-n$	0	0	0	$-\ell$
$U(1)_R$	0	0	0	0	0	0

	$(\chi_{15}, \chi_{16}, \chi_{17})$	χ_1^0	χ_2^0	χ_3^0	(χ_4^0, χ_5^0)
$SU(5)$	1	1	1	1	1
S_4	3	1	1	1	2
Z_4	-1	-1	i	-1	$-i$
$U(1)_{FN}$	$-z$	$2\ell + n$	0	2ℓ	z
$U(1)_R$	0	2	2	2	2

Table 2: Assignments of $SU(5)$, S_4 , Z_4 , $U(1)_{FN}$, and $U(1)_R$ representations for flavons and driving fields.

Three S_4 singlets χ_1^0 , χ_2^0 , and χ_3^0 and one S_4 doublet (χ_4^0, χ_5^0) for driving fields are required to obtain relevant vacuum alignment in our model. Moreover, an S_4 triplet $(\chi_{15}, \chi_{16}, \chi_{17})$ is

introduced as additional flavons². Assignments of flavons and driving fields are summarized in Table 2.

The $SU(5) \times S_4 \times Z_4 \times U(1)_{FN} \times U(1)_R$ invariant superpotential is given as

$$\begin{aligned}
w' = & \kappa_1 (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes (\chi_3, \chi_4) \otimes \chi_1^0 / \Lambda \\
& + \eta_1 (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_2^0 \\
& + \eta_2 (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes \chi_3^0 + \eta_3 \chi_{14} \otimes \chi_{14} \otimes \chi_3^0 \\
& + \eta_4 (\chi_5, \chi_6, \chi_7) \otimes (\chi_{15}, \chi_{16}, \chi_{17}) \otimes (\chi_4^0, \chi_5^0),
\end{aligned} \tag{67}$$

which is rewritten as

$$\begin{aligned}
w' = & \kappa_1 [2\chi_1\chi_2\chi_3 + (\chi_1^2 - \chi_2^2)\chi_4] \chi_1^0 / \Lambda + \eta_1 (\chi_8\chi_{11} + \chi_9\chi_{12} + \chi_{10}\chi_{13}) \chi_2^0 \\
& + [\eta_2(\chi_1^2 + \chi_2^2) + \eta_3\chi_{14}^2] \chi_3^0 + \frac{1}{\sqrt{2}}\eta_4 (\chi_6\chi_{16} - \chi_7\chi_{17}) \chi_4^0 \\
& + \frac{1}{\sqrt{6}}\eta_4 (-2\chi_5\chi_{15} + \chi_6\chi_{16} + \chi_7\chi_{17}) \chi_5^0.
\end{aligned} \tag{68}$$

Then the scalar potential is given as

$$\begin{aligned}
V = & \left| \frac{\kappa_1}{\Lambda} [2\chi_1\chi_2\chi_3 + (\chi_1^2 - \chi_2^2)\chi_4] \right|^2 + |\eta_1 (\chi_8\chi_{11} + \chi_9\chi_{12} + \chi_{10}\chi_{13})|^2 \\
& + |\eta_2(\chi_1^2 + \chi_2^2) + \eta_3\chi_{14}^2|^2 + \left| \frac{1}{\sqrt{2}}\eta_4 (\chi_6\chi_{16} - \chi_7\chi_{17}) \right|^2 \\
& + \left| \frac{1}{\sqrt{6}}\eta_4 (-2\chi_5\chi_{15} + \chi_6\chi_{16} + \chi_7\chi_{17}) \right|^2.
\end{aligned} \tag{69}$$

Therefore conditions to realize the potential minimum ($V = 0$) are given as

$$\begin{aligned}
& \kappa_1 [2\chi_1\chi_2\chi_3 + (\chi_1^2 - \chi_2^2)\chi_4] / \Lambda = 0, \\
& \eta_1 (\chi_8\chi_{11} + \chi_9\chi_{12} + \chi_{10}\chi_{13}) = 0, \\
& \eta_2(\chi_1^2 + \chi_2^2) + \eta_3\chi_{14}^2 = 0, \\
& \frac{1}{\sqrt{2}}\eta_4 (\chi_6\chi_{16} - \chi_7\chi_{17}) = 0, \\
& \frac{1}{\sqrt{6}}\eta_4 (-2\chi_5\chi_{15} + \chi_6\chi_{16} + \chi_7\chi_{17}) = 0,
\end{aligned} \tag{70}$$

where χ_i 's are regarded as VEVs. One of the solution which satisfies these conditions is obtained as

$$\begin{aligned}
\chi_1 = \chi_2, \quad \chi_3 = 0, \quad \chi_5 = \chi_6 = \chi_7, \quad \chi_8 = \chi_{10} = \chi_{11} = \chi_{12} = 0, \\
\chi_{14}^2 = -\frac{2\eta_2}{\eta_3}\chi_1^2, \quad \chi_{15} = \chi_{16} = \chi_{17}.
\end{aligned} \tag{71}$$

Therefore we obtain the desired alignment of VEVs in Eq. (64). Next-to-leading couplings of flavons and driving fields could shift these alignments. Detail discussions are given in Appendix C.

²As far as $z \gg 1$, $(\chi_{15}, \chi_{16}, \chi_{17})$ do not disturb the result in section 2.

4 Soft SUSY breaking terms

We have already discussed SUSY breaking terms i.e., sfermion masses and scalar trilinear couplings in the D_4 flavor model [104, 105], the A_4 flavor model [49], and the $\Delta(54)$ flavor model [106, 107]. In this section, we study SUSY breaking terms in the framework of $S_4 \times Z_4 \times U(1)_{FN}$. We consider the gravity mediation within the framework of supergravity theory. We assume that non-vanishing F -terms of gauge and flavor singlet (moduli) fields Z and gauge singlet fields χ_i ($i = 1, \dots, 14$) contribute to the SUSY breaking. Their F -components are written as

$$F^{\Phi_k} = -e^{\frac{K}{2M_p^2}} K^{\Phi_k \bar{I}} \left(\partial_{\bar{I}} \bar{W} + \frac{K_{\bar{I}}}{M_p^2} \bar{W} \right), \quad (72)$$

where K denotes the Kähler potential, $K_{\bar{I}J}$ denotes second derivatives by fields, i.e. $K_{\bar{I}J} = \partial_{\bar{I}} \partial_J K$ and $K^{\bar{I}J}$ is its inverse. Here the fields Φ_k correspond to the moduli fields Z and gauge singlet fields χ_i . The VEVs of F_{Φ_k}/Φ_k are estimated as $\langle F_{\Phi_k}/\Phi_k \rangle = \mathcal{O}(m_{3/2})$, where $m_{3/2}$ denotes the gravitino mass, which is obtained as $m_{3/2} = \langle e^{K/2M_p^2} W/M_p^2 \rangle$.

4.1 Soft scalar masses of slepton sector

First, let us study soft scalar masses. Within the framework of supergravity theory, soft scalar mass squared is obtained as [108]

$$m_{\bar{I}J}^2 K_{\bar{I}J} = m_{3/2}^2 K_{\bar{I}J} + |F^{\Phi_k}|^2 \partial_{\Phi_k} \partial_{\bar{\Phi}_k} K_{\bar{I}J} - |F^{\Phi_k}|^2 \partial_{\bar{\Phi}_k} K_{\bar{I}L} \partial_{\Phi_k} K_{\bar{M}J} K^{LM}. \quad (73)$$

The invariance under the $S_4 \times Z_4 \times U(1)_{FN}$ flavor symmetry as well as the gauge invariance requires the following form of the Kähler potential as

$$K = Z^{(L)}(\Phi) \sum_{i=e,\mu,\tau} |L_i|^2 + Z_{(1)}^{(R)}(\Phi) \sum_{i=e,\mu} |R_i|^2 + Z_{(2)}^{(R)}(\Phi) |R_\tau|^2, \quad (74)$$

at the lowest level, where $Z^{(L)}(\Phi)$ and $Z_{(1),(2)}^{(R)}(\Phi)$ are arbitrary functions of the singlet fields Φ . By use of Eq. (73) with the Kähler potential in Eq. (74), we obtain the following matrix form of soft scalar masses squared for left-handed and right-handed charged sleptons,

$$(m_{\bar{L}}^2)_{ij} = \begin{pmatrix} m_L^2 & 0 & 0 \\ 0 & m_L^2 & 0 \\ 0 & 0 & m_L^2 \end{pmatrix}, \quad (m_{\bar{R}}^2)_{ij} = \begin{pmatrix} m_{R(1)}^2 & 0 & 0 \\ 0 & m_{R(1)}^2 & 0 \\ 0 & 0 & m_{R(2)}^2 \end{pmatrix}. \quad (75)$$

That is, three left-handed slepton masses are degenerate, and two right-handed slepton masses are degenerate. These predictions would be obvious because the left-handed sleptons form a triplet of S_4 , and the right-handed slepton form a doublet and a singlet of S_4 . These predictions hold exactly before $S_4 \times Z_4 \times U(1)_{FN}$ is broken, but its breaking gives next-to-leading terms in the slepton mass matrices.

Next, we study effects due to $S_4 \times Z_4 \times U(1)_{FN}$ breaking by χ_i . That is, we estimate corrections to the Kähler potential including χ_i . Since each VEV is taken as the same order, the breaking scale can be characterized by the average of VEVs, such as $\tilde{\alpha}\Lambda$.

In our model, the right-handed charged leptons (R_e^c, R_μ^c) are assigned to **2** and its conjugate representation is itself **2**. Similarly, the left-handed charged leptons (L_e, L_μ, L_τ) are assigned to **3** and its conjugation is **3**. Therefore, for left-handed sector, higher dimensional terms are given as

$$\begin{aligned}\Delta K_L = & \sum_{i=1,3} Z_{\Delta_{a_i}}^{(L)}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L_e^c, L_\mu^c, L_\tau^c) \otimes (\chi_i, \chi_{i+1}) \otimes (\chi_i^c, \chi_{i+1}^c)/\Lambda^2 \\ & + \sum_{i=5,8,11} Z_{\Delta_{b_i}}^{(L)}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L_e^c, L_\mu^c, L_\tau^c) \otimes (\chi_i, \chi_{i+1}, \chi_{i+2}) \otimes (\chi_i^c, \chi_{i+1}^c, \chi_{i+2}^c)/\Lambda^2 \\ & + Z_{\Delta_c}^{(L)}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L_e^c, L_\mu^c, L_\tau^c) \otimes \chi_{14} \otimes \chi_{14}^c/\Lambda^2 \\ & + Z_{\Delta_d}^{(L)}(\Phi)(L_e, L_\mu, L_\tau) \otimes (L_e^c, L_\mu^c, L_\tau^c) \otimes \Theta \otimes \Theta^c/\bar{\Lambda}^2.\end{aligned}\quad (76)$$

For example, higher dimensional terms including (χ_1, χ_2) and (χ_5, χ_6, χ_7) are explicitly written as

$$\begin{aligned}\Delta K_L^{[\chi_1, \chi_5]} = & Z_{\Delta_{a_1}}^{(L)}(\Phi) \left[\frac{\sqrt{2}|\chi_1|^2}{\Lambda^2} (|L_\mu|^2 - |L_\tau|^2) \right] \\ & + Z_{\Delta_{b_5}}^{(L)}(\Phi) \left[\frac{2|\chi_5|^2}{\Lambda^2} (L_\mu L_\tau^* + L_\tau L_\mu^* + L_e L_\tau^* + L_\tau L_e^* + L_e L_\mu^* + L_\mu L_e^*) \right].\end{aligned}\quad (77)$$

When we take into account the corrections from all $\chi_i \chi_j^*$ to the Kähler potential, the soft scalar masses squared for left-handed charged sleptons have the following corrections,

$$(m_L^2)_{ij} = \begin{pmatrix} m_L^2 + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) \\ \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & m_L^2 + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) \\ \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & m_L^2 + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) \end{pmatrix}, \quad (78)$$

where $\tilde{\alpha}$ is a linear combination of α_i 's.

For right-handed sector, higher dimensional terms are given as

$$\begin{aligned}\Delta K_R = & \sum_{i=1,3} Z_{\Delta_{a_i}}^{(R)}(\Phi)(R_e, R_\mu) \otimes (R_e^c, R_\mu^c) \otimes (\chi_i, \chi_{i+1}) \otimes (\chi_i^c, \chi_{i+1}^c)/\Lambda^2 \\ & + \sum_{i=5,8,11} Z_{\Delta_{b_i}}^{(R)}(\Phi)(R_e, R_\mu) \otimes (R_e^c, R_\mu^c) \otimes (\chi_i, \chi_{i+1}, \chi_{i+2}) \otimes (\chi_i^c, \chi_{i+1}^c, \chi_{i+2}^c)/\Lambda^2 \\ & + Z_{\Delta_c}^{(R)}(\Phi)(R_e, R_\mu) \otimes (R_e^c, R_\mu^c) \otimes \chi_{14} \otimes \chi_{14}^c/\Lambda^2 \\ & + Z_{\Delta_d}^{(R)}(\Phi)(R_e, R_\mu) \otimes R_\tau^c \otimes (\chi_1, \chi_2)/\Lambda^2 + Z_{\Delta_e}^{(R)}(\Phi)(R_e^c, R_\mu^c) \otimes R_\tau \otimes (\chi_1^c, \chi_2^c)/\Lambda^2 \\ & + \sum_{i=1,3} Z_{\Delta_{f_i}}^{(R)}(\Phi) R_\tau \otimes R_\tau^c \otimes (\chi_i, \chi_{i+1}) \otimes (\chi_i^c, \chi_{i+1}^c)/\Lambda^2 \\ & + \sum_{i=5,8,11} Z_{\Delta_{g_i}}^{(R)}(\Phi) R_\tau \otimes R_\tau^c \otimes (\chi_i, \chi_{i+1}, \chi_{i+2}) \otimes (\chi_i^c, \chi_{i+1}^c, \chi_{i+2}^c)/\Lambda^2 \\ & + Z_{\Delta_h}^{(R)}(\Phi) R_\tau \otimes R_\tau^c \otimes \chi_{14} \otimes \chi_{14}^c/\Lambda^2 \\ & + Z_{\Delta_i}^{(R)}(\Phi)(R_e, R_\mu) \otimes (R_e^c, R_\mu^c) \otimes \Theta \otimes \Theta^c/\bar{\Lambda}^2 \\ & + Z_{\Delta_j}^{(R)}(\Phi) R_\tau \otimes R_\tau^c \otimes \Theta \otimes \Theta^c/\bar{\Lambda}^2.\end{aligned}\quad (79)$$

In the same way, right-handed charged sleptons can be written as

$$(m_{\tilde{R}}^2)_{ij} = \begin{pmatrix} m_{R(1)}^2 + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & \mathcal{O}(\alpha_1 m_{3/2}^2) \\ \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & m_{R(1)}^2 + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & \mathcal{O}(\alpha_1 m_{3/2}^2) \\ \mathcal{O}(\alpha_1 m_{3/2}^2) & \mathcal{O}(\alpha_1 m_{3/2}^2) & m_{R(2)}^2 + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) \end{pmatrix}. \quad (80)$$

In order to estimate the magnitude of FCNC, we move to super-CKM basis by diagonalizing the charged lepton mass matrix including next-to-leading terms. For the left-handed slepton mass matrix, we obtain

$$\begin{aligned} (m_{\tilde{L}}^2)^{(SCKM)}_{ij} &= U_E^\dagger (m_L^2)_{ij} U_E \\ &\simeq \begin{pmatrix} m_L^2 + \mathcal{O}(\frac{\tilde{\alpha}^2}{\lambda^2} m_{3/2}^2) & \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) \\ \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & m_L^2 + \mathcal{O}(\frac{\tilde{\alpha}^2}{\lambda^2} m_{3/2}^2) & \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) \\ \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & m_L^2 + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) \end{pmatrix}, \end{aligned} \quad (81)$$

where we take the left-handed mixing

$$U_E = \begin{pmatrix} 1 & \frac{\tilde{\alpha}}{\lambda} & \tilde{\alpha} \\ -\frac{\tilde{\alpha}}{\lambda} - \tilde{\alpha}^2 & 1 & \tilde{\alpha} \\ -\tilde{\alpha} + \frac{\tilde{\alpha}^2}{\lambda} & -\tilde{\alpha} - \frac{\tilde{\alpha}^2}{\lambda} & 1 \end{pmatrix}. \quad (82)$$

In Eq. (82), the unitarity is satisfied up to $\mathcal{O}(\tilde{\alpha}^2)$.

For the right-handed slepton mass matrix, we obtain

$$\begin{aligned} (m_{\tilde{R}}^2)^{(SCKM)}_{ij} &= V_E^\dagger (m_R^2)_{ij} V_E \\ &\simeq \begin{pmatrix} m_{R(1)}^2 + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & \mathcal{O}(\alpha_1 m_{3/2}^2) \\ \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & m_{R(1)}^2 + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) & \mathcal{O}(\alpha_1 m_{3/2}^2) \\ \mathcal{O}(\alpha_1 m_{3/2}^2) & \mathcal{O}(\alpha_1 m_{3/2}^2) & m_{R(2)}^2 + \mathcal{O}(\tilde{\alpha}^2 m_{3/2}^2) \end{pmatrix}, \end{aligned} \quad (83)$$

where we take the right-handed mixing as

$$V_E = \begin{pmatrix} \cos 15^\circ & \sin 15^\circ & 0 \\ -\sin 15^\circ & \cos 15^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1 & \frac{\tilde{\alpha}^2}{\lambda^2} & \tilde{\alpha} \\ -\frac{\tilde{\alpha}^2}{\lambda^2} - \tilde{\alpha}^2 & 1 & \tilde{\alpha} \\ -\tilde{\alpha} + \frac{\tilde{\alpha}^3}{\lambda^2} & -\tilde{\alpha} - \frac{\tilde{\alpha}^3}{\lambda^2} & 1 \end{pmatrix}. \quad (84)$$

In Eq. (84), the unitarity is satisfied up to $\mathcal{O}(\tilde{\alpha}^2)$.

Off-diagonal entries of $(m_{\tilde{L}}^2)^{(SCKM)}_{ij}$ and $(m_{\tilde{R}}^2)^{(SCKM)}_{ij}$ are constrained by the FCNC experiments [109]. Our model predicts

$$(\Delta_{LL})_{12} \equiv \frac{(m_{\tilde{L}}^2)^{(SCKM)}_{12}}{(m_{\tilde{L}}^2)_{11}} = \mathcal{O}(\tilde{\alpha}^2), \quad (\Delta_{RR})_{12} \equiv \frac{(m_{\tilde{R}}^2)^{(SCKM)}_{12}}{(m_{\tilde{R}}^2)_{11}} = \mathcal{O}(\tilde{\alpha}^2), \quad (85)$$

where we take $m_L^2 = m_{R(1)}^2 = m_{3/2}^2$. The $\mu \rightarrow e\gamma$ experiment [110] constrains these values as $(\Delta_{LL})_{12}^{\text{exp}}, (\Delta_{RR})_{12}^{\text{exp}} \leq \mathcal{O}(10^{-3})$ [109], when $m_{\tilde{L}} \simeq m_{\tilde{R}} \simeq 100$ GeV. On the other hand, the parameter space in the previous section corresponds to $\tilde{\alpha} \simeq 10^{-2}$ and gives $(\Delta_{LL})_{12}, (\Delta_{RR})_{12} \leq \mathcal{O}(10^{-4})$. Thus, our parameter region is favored from the viewpoint of the FCNC constraint.

4.2 A-term of slepton sector

Here, let us study scalar trilinear couplings, i.e. the so called A-terms. The A-terms among left-handed and right-handed sleptons and Higgs scalar fields are obtained in the gravity mediation as [108]

$$h_{IJ}L_JR_IH_K = \sum_{K=\bar{5}, 45} h_{IJK}^{(Y)}L_JR_IH_K + h_{IJK}^{(K)}L_JR_IH_K, \quad (86)$$

where

$$\begin{aligned} h_{IJK}^{(Y)} &= F^{\Phi_k} \langle \partial_{\Phi_k} \tilde{y}_{IJK} \rangle, \\ h_{IJK}^{(K)}L_JR_IH_K &= -\langle \tilde{y}_{LJK} \rangle L_JR_IH_K F^{\Phi_k} K^{L\bar{L}} \partial_{\Phi_k} K_{\bar{L}I} \\ &\quad -\langle \tilde{y}_{IMK} \rangle L_JR_IH_K F^{\Phi_k} K^{M\bar{M}} \partial_{\Phi_k} K_{\bar{M}J} \\ &\quad -\langle \tilde{y}_{IJK} \rangle L_JR_IH_K F^{\Phi_k} K^{H_d} \partial_{\Phi_k} K_{H_K}, \end{aligned} \quad (87)$$

and K_{H_K} denotes the Kähler metric of H_K . In addition, \tilde{y}_{IJK} denotes effective Yukawa couplings, and it corresponds to

$$\tilde{y}_{IJK} = -3y_1 \begin{pmatrix} 0 & \alpha_9/\sqrt{2} & -\alpha_{10}/\sqrt{2} \\ -2\alpha_8/\sqrt{6} & \alpha_9/\sqrt{6} & \alpha_{10}/\sqrt{6} \\ 0 & 0 & 0 \end{pmatrix} + y_2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \alpha_{11} & \alpha_{12} & \alpha_{13} \end{pmatrix}. \quad (88)$$

Since the vacuum alignment indicates $\alpha_{10} = \alpha_{12} = \alpha_{13} = \alpha_{14} = 0$, we get

$$\tilde{y}_{IJK} = -3y_1 \begin{pmatrix} 0 & \alpha_9/\sqrt{2} & 0 \\ 0 & \alpha_9/\sqrt{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} + y_2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha_{13} \end{pmatrix}, \quad (89)$$

then

$$h_{IJK}^{(Y)} = -\frac{3y_1}{\Lambda} \begin{pmatrix} 0 & \tilde{F}^{\alpha_9}/\sqrt{2} & 0 \\ 0 & \tilde{F}^{\alpha_9}/\sqrt{6} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{y_2}{\Lambda} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \tilde{F}^{\alpha_{13}} \end{pmatrix}, \quad (90)$$

where $\tilde{F}^{\alpha_i} = F^{\alpha_i}/\alpha_i$ and $\tilde{F}^{\alpha_i}/\Lambda = \mathcal{O}(m_{3/2})$.

Next, we estimate $h_{IJK}^{(K)}$. When we neglect correction terms and use the lowest level of Kähler potential, we obtain

$$h_{IJK}^{(K)} = \tilde{y}_{IJK}(A_I^R + A_J^L), \quad (91)$$

where we estimate $A_1^L = A_2^L = A_3^L = F^{\tilde{\alpha}_i}/(\alpha_i\Lambda) \simeq \mathcal{O}(m_{3/2})$. The magnitudes of $A_1^R = A_2^R$ and A_3^R are also $\mathcal{O}(m_{3/2})$.

Furthermore, we should take into account next-to-leading terms of the Kähler potential including χ_i . These correction terms appear all entries so that their magnitudes are suppressed in $\mathcal{O}(\tilde{\alpha})$ compared with the leading term. Then, we obtain

$$(m_{LR}^2)_{ij} \simeq m_{3/2} \begin{pmatrix} \mathcal{O}(\tilde{\alpha}^2 v_d) & \frac{\sqrt{3}m_\mu}{2} & \mathcal{O}(\tilde{\alpha}^2 v_d) \\ \mathcal{O}(\tilde{\alpha}^2 v_d) & \frac{m_\mu}{2} & \mathcal{O}(\tilde{\alpha}^2 v_d) \\ \mathcal{O}(\tilde{\alpha}^2 v_d) & \mathcal{O}(\tilde{\alpha}^2 v_d) & \mathcal{O}(m_\tau) \end{pmatrix}. \quad (92)$$

Moving to the super-CKM basis, we have

$$\begin{aligned}
(m_{LR}^2)_{ij}^{SCKM} &= U_E^\dagger (m_{LR}^2)_{ij} V_E \simeq m_{3/2} \begin{pmatrix} \mathcal{O}(\tilde{\alpha}^2 v_d) & \mathcal{O}(\tilde{\alpha}^2 v_d) & \mathcal{O}(\tilde{\alpha}^2 v_d) \\ \mathcal{O}(\tilde{\alpha}^2 v_d) & \mathcal{O}(m_\mu) & \mathcal{O}(\tilde{\alpha}^2 v_d) \\ \mathcal{O}(\tilde{\alpha}^2 v_d) & \mathcal{O}(\tilde{\alpha}^2 v_d) & \mathcal{O}(m_\tau) \end{pmatrix} \\
&\simeq m_{3/2} \begin{pmatrix} \mathcal{O}(m_e) & \mathcal{O}(m_e) & \mathcal{O}(m_e) \\ \mathcal{O}(m_e) & \mathcal{O}(m_\mu) & \mathcal{O}(m_e) \\ \mathcal{O}(m_e) & \mathcal{O}(m_e) & \mathcal{O}(m_\tau) \end{pmatrix}.
\end{aligned} \tag{93}$$

The FCNC measure

$$(\Delta_{LR})_{12} \equiv \frac{(\tilde{m}_{LR}^2)_{12}}{m_{3/2}^2} = \frac{\mathcal{O}(m_e)}{m_{3/2}}, \tag{94}$$

is predicted to be of order 5×10^{-6} for $m_{3/2} = 100$ GeV.

This ratio is marginal compared with the bound $\mathcal{O}(10^{-6})$ by the $\mu \rightarrow e\gamma$ experiments when the slepton mass is 100 GeV. Therefore, we expect that the $\mu \rightarrow e\gamma$ process will be observed in the near future if slepton mass is 100 GeV.

4.3 FCNC from neutrino sector

Next, we consider the contribution on FCNC from the neutrino sector. The typical process of the FCNC is the $\mu \rightarrow e\gamma$ [110]. The key quantity is the magnitude of $(Y_D^\dagger Y_D)_{12}$, which is given in the following matrix:

$$Y_D^\dagger Y_D = \frac{\alpha_5^2}{3} \begin{pmatrix} 3y_2^{D^2} + 2y_1^{D^2}\lambda^2 & 3y_2^{D^2} - y_1^{D^2}\lambda^2 & 3y_2^{D^2} - y_1^{D^2}\lambda^2 \\ 3y_2^{D^2} - y_1^{D^2}\lambda^2 & 3y_2^{D^2} + 2y_1^{D^2}\lambda^2 & 3y_2^{D^2} - y_1^{D^2}\lambda^2 \\ 3y_2^{D^2} - y_1^{D^2}\lambda^2 & 3y_2^{D^2} - y_1^{D^2}\lambda^2 & 3y_2^{D^2} + 2y_1^{D^2}\lambda^2 \end{pmatrix} \simeq y_2^{D^2} \begin{pmatrix} \alpha_5^2 & \alpha_5^2 & \alpha_5^2 \\ \alpha_5^2 & \alpha_5^2 & \alpha_5^2 \\ \alpha_5^2 & \alpha_5^2 & \alpha_5^2 \end{pmatrix}. \tag{95}$$

The FCNC measure on $\mu \rightarrow e\gamma$ is calculated as follows [111, 112, 113]:

$$(\Delta_{LL})_{12} \equiv \frac{(\Delta m)_{12}^2}{M_{\text{SUSY}}^2} = \frac{6m_0^2}{16\pi^2 M_{\text{SUSY}}^2} (Y_D^\dagger Y_D)_{12} \ln \frac{\Lambda}{M} \simeq \frac{3}{8\pi^2} y_2^{D^2} \alpha_5^2 \ln \frac{\Lambda}{M} \simeq 6 \times 10^{-5}, \tag{96}$$

where we put $m_0 = M_{\text{SUSY}}$, $\alpha_5 = 10^{-2}$, $\Lambda = 10^{18}$ GeV, $M = 10^{12}$ GeV. It is concluded that the contribution on $\mu \rightarrow e\gamma$ from the neutrino sector is much smaller than the experimental bound $(\Delta_{LL})_{12}^{\text{exp}} \leq \mathcal{O}(10^{-3})$ [109].

5 Summary

We have presented a flavor model with the S_4 symmetry to unify quarks and leptons in the framework of the $SU(5)$ SUSY GUT. Three generations of $\bar{\mathbf{5}}$ -plets in $SU(5)$ are assigned to $\mathbf{3}$ of S_4 while the first and second generations of 10-plets in $SU(5)$ are assigned to $\mathbf{2}$ of S_4 , and the third generation of 10-plet is assigned to $\mathbf{1}$ of S_4 . These assignments of S_4 for $\bar{\mathbf{5}}$ and 10 lead to the completely different structure of quark and lepton mass matrices. Right-handed

neutrinos, which are $SU(5)$ gauge singlets, are also assigned to $\mathbf{2}$ for the first and second generations and $\mathbf{1}'$ for the third generation. These assignments realize the tri-bimaximal mixing of neutrino flavors. The vacuum alignment of scalars is also required to realize the tri-bimaximal mixing of neutrino flavors. Our model predicts the quark mixing as well as the tri-bimaximal mixing of leptons. Especially, the Cabibbo angle is predicted to be around 15° . Our model is consistent with observed CKM mixing angles and CP violation as well as the non-vanishing U_{e3} of the neutrino flavor mixing. The deviation from 15° in $|V_{us}^0|$ is given by $\mathcal{O}(m_d/m_s)$. Therefore, we can adjust one parameter at the next-to-leading order to reproduce the observed Cabibbo angle. The non-vanishing U_{e3} of the neutrino flavor mixing is also predicted to be ~ 0.02 .

We have also studied SUSY breaking terms. In our model, three families of left-handed slepton masses are degenerate and two right-handed sleptons are degenerate. Even although we take into account corrections due to the flavor symmetry breaking, our model leads to marginal values of FCNC's compared with the present experimental bounds. Therefore, we expect the observation of the $\mu \rightarrow e\gamma$ process in the near future.

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Appendix

A Multiplication rule of S_4

The S_4 group has 24 distinct elements and irreducible representations $\mathbf{1}$, $\mathbf{1}'$, $\mathbf{2}$, $\mathbf{3}$, and $\mathbf{3}'$. The multiplication rule depends on the basis. One can see its basis dependence in our review [9]. In this appendix, we present the multiplication rule, which is used in this paper:

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}_{\mathbf{2}} = (a_1b_1 + a_2b_2)_{\mathbf{1}} \oplus (-a_1b_2 + a_2b_1)_{\mathbf{1}'} \oplus \begin{pmatrix} a_1b_2 + a_2b_1 \\ a_1b_1 - a_2b_2 \end{pmatrix}_{\mathbf{2}}, \quad (97)$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} = \begin{pmatrix} a_2b_1 \\ -\frac{1}{2}(\sqrt{3}a_1b_2 + a_2b_2) \\ \frac{1}{2}(\sqrt{3}a_1b_3 - a_2b_3) \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_1b_1 \\ \frac{1}{2}(\sqrt{3}a_2b_2 - a_1b_2) \\ -\frac{1}{2}(\sqrt{3}a_2b_3 + a_1b_3) \end{pmatrix}_{\mathbf{3}'}, \quad (98)$$

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}'} = \begin{pmatrix} a_1b_1 \\ \frac{1}{2}(\sqrt{3}a_2b_2 - a_1b_2) \\ -\frac{1}{2}(\sqrt{3}a_2b_3 + a_1b_3) \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_2b_1 \\ -\frac{1}{2}(\sqrt{3}a_1b_2 + a_2b_2) \\ \frac{1}{2}(\sqrt{3}a_1b_3 - a_2b_3) \end{pmatrix}_{\mathbf{3}'}, \quad (99)$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} = (a_1b_1 + a_2b_2 + a_3b_3)_{\mathbf{1}} \oplus \begin{pmatrix} \frac{1}{\sqrt{2}}(a_2b_2 - a_3b_3) \\ \frac{1}{\sqrt{6}}(-2a_1b_1 + a_2b_2 + a_3b_3) \end{pmatrix}_{\mathbf{2}} \\ \oplus \begin{pmatrix} a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_3b_2 - a_2b_3 \\ a_1b_3 - a_3b_1 \\ a_2b_1 - a_1b_2 \end{pmatrix}_{\mathbf{3}'}, \quad (100)$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}'} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}'} = (a_1b_1 + a_2b_2 + a_3b_3)_{\mathbf{1}} \oplus \begin{pmatrix} \frac{1}{\sqrt{2}}(a_2b_2 - a_3b_3) \\ \frac{1}{\sqrt{6}}(-2a_1b_1 + a_2b_2 + a_3b_3) \end{pmatrix}_{\mathbf{2}} \\ \oplus \begin{pmatrix} a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_3b_2 - a_2b_3 \\ a_1b_3 - a_3b_1 \\ a_2b_1 - a_1b_2 \end{pmatrix}_{\mathbf{3}'}, \quad (101)$$

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}'} = (a_1b_1 + a_2b_2 + a_3b_3)_{\mathbf{1}'} \oplus \begin{pmatrix} \frac{1}{\sqrt{6}}(2a_1b_1 - a_2b_2 - a_3b_3) \\ \frac{1}{\sqrt{2}}(a_2b_2 - a_3b_3) \end{pmatrix}_{\mathbf{2}} \\ \oplus \begin{pmatrix} a_3b_2 - a_2b_3 \\ a_1b_3 - a_3b_1 \\ a_2b_1 - a_1b_2 \end{pmatrix}_{\mathbf{3}} \oplus \begin{pmatrix} a_2b_3 + a_3b_2 \\ a_1b_3 + a_3b_1 \\ a_1b_2 + a_2b_1 \end{pmatrix}_{\mathbf{3}'}. \quad (102)$$

More details are shown in the review [9].

B Determination of ℓ , m , and n

The charged lepton masses in Eq. (11) give

$$\alpha_9 = \frac{m_\mu}{\sqrt{6}|\bar{y}_1|\lambda^\ell v_d} . \quad (103)$$

Therefore, if we take $|\bar{y}_1| = 1$, $\lambda = 0.1$, and $m_\mu = 6.86 \times 10^{-2}$ GeV, we get

$$\alpha_9 \sim 5.1 \times 10^{\ell-4} . \quad (104)$$

Suppose that magnitudes of all α_i are same order 10^{-2} as seen in Eq. (66). So we take $\ell = 1$, which gives $\alpha_9 \sim 5.1 \times 10^{-3}$.

We take the right-handed neutrino mass M as follows:

$$M = \mathcal{O}(10^{12}) \text{ GeV} . \quad (105)$$

As seen in Eqs. (15) and (17), parameters a and c should be comparable. Therefore, we have

$$y_2^N \lambda^{-n} \alpha_4 \Lambda \sim M = \mathcal{O}(10^{12}) \text{ GeV} . \quad (106)$$

Again suppose that magnitudes of all α_i are same order 10^{-2} , we get following equation as

$$y_2^N \Lambda \sim \mathcal{O}(10^{14-n}) . \quad (107)$$

where $\lambda = 0.1$. On the other hand, m and n satisfy the condition:

$$0 < m < n \leq 2m . \quad (108)$$

Therefore, we have the smallest value of n which satisfies Eq. (108) as

$$m = 1, \quad n = 2 . \quad (109)$$

C Next-to-leading terms of the scalar potential

We consider the next-to-leading terms of the scalar potential. In the next-to-leading order, the $SU(5) \times S_4 \times Z_4 \times U(1)_{FN} \times U(1)_R$ invariant operators which couple to driving fields are given as follows;

- Coupled with χ_1^0 :

$$\begin{aligned} & (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes \chi_1^0 \otimes \Theta^n / \bar{\Lambda}^n, \\ & \chi_{14} \otimes \chi_{14} \otimes \chi_1^0 \otimes \Theta^n / \bar{\Lambda}^n, \\ & (\chi_5, \chi_6, \chi_7) \otimes (\chi_5, \chi_6, \chi_7) \otimes \chi_1^0 \otimes \Theta^{2\ell+n} / \bar{\Lambda}^{2\ell+n}, \\ & (\chi_{11}, \chi_{12}, \chi_{13}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_1^0 \otimes \Theta^{2\ell+n} / \bar{\Lambda}^{2\ell+n}, \\ & (\chi_3, \chi_4) \otimes (\chi_5, \chi_6, \chi_7) \otimes (\chi_5, \chi_6, \chi_7) \otimes \chi_1^0 \otimes \Theta^{2\ell} / (\Lambda \bar{\Lambda}^{2\ell}), \\ & (\chi_3, \chi_4) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_1^0 \otimes \Theta^{2\ell} / (\Lambda \bar{\Lambda}^{2\ell}), \\ & (\chi_5, \chi_6, \chi_7) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_1^0 \otimes \Theta^{2\ell+n} / (\Lambda \bar{\Lambda}^{2\ell+n}), \\ & (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes \chi_1^0 \otimes \Theta^{2\ell+n} / (\Lambda \bar{\Lambda}^{2\ell+n}). \end{aligned} \quad (110)$$

- Coupled with χ_2^0 :

$$\begin{aligned}
& (\chi_5, \chi_6, \chi_7) \otimes (\chi_5, \chi_6, \chi_7) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_2^0/\Lambda, \\
& (\chi_5, \chi_6, \chi_7) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes \chi_2^0/\Lambda, \\
& (\chi_{11}, \chi_{12}, \chi_{13}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_2^0/\Lambda.
\end{aligned} \tag{111}$$

- Coupled with χ_3^0 :

$$\begin{aligned}
& (\chi_5, \chi_6, \chi_7) \otimes (\chi_5, \chi_6, \chi_7) \otimes \chi_3^0 \otimes \Theta^{2\ell}/\bar{\Lambda}^{2\ell}, \\
& (\chi_{11}, \chi_{12}, \chi_{13}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_3^0 \otimes \Theta^{2\ell}/\bar{\Lambda}^{2\ell}, \\
& (\chi_3, \chi_4) \otimes (\chi_5, \chi_6, \chi_7) \otimes (\chi_5, \chi_6, \chi_7) \otimes \chi_3^0 \otimes \Theta^{2\ell-n}/(\Lambda\bar{\Lambda}^{2\ell-n}), \\
& (\chi_3, \chi_4) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_3^0 \otimes \Theta^{2\ell-n}/(\Lambda\bar{\Lambda}^{2\ell-n}), \\
& (\chi_5, \chi_6, \chi_7) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes \chi_3^0 \otimes \Theta^{2\ell}/(\Lambda\bar{\Lambda}^{2\ell}), \\
& (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes \chi_3^0 \otimes \Theta^{2\ell}/(\Lambda\bar{\Lambda}^{2\ell}).
\end{aligned} \tag{112}$$

- Coupled with (χ_4^0, χ_5^0) :

$$\begin{aligned}
& (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes (\chi_{15}, \chi_{16}, \chi_{17}) \otimes (\chi_4^0, \chi_5^0)/\Lambda, \\
& (\chi_5, \chi_6, \chi_7) \otimes (\chi_5, \chi_6, \chi_7) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes (\chi_{15}, \chi_{16}, \chi_{17}) \otimes (\chi_4^0, \chi_5^0)/\Lambda^2, \\
& (\chi_5, \chi_6, \chi_7) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_8, \chi_9, \chi_{10}) \otimes (\chi_{15}, \chi_{16}, \chi_{17}) \otimes (\chi_4^0, \chi_5^0)/\Lambda^2, \\
& (\chi_{11}, \chi_{12}, \chi_{13}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes (\chi_{11}, \chi_{12}, \chi_{13}) \otimes (\chi_{15}, \chi_{16}, \chi_{17}) \otimes (\chi_4^0, \chi_5^0)/\Lambda^2.
\end{aligned} \tag{113}$$

As seen in Eqs. (67) and (110), we can write the superpotential which couples to χ_1^0 as

$$\begin{aligned}
w_{\chi_1^0} &= \kappa_1 (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes (\chi_3, \chi_4) \otimes \chi_1^0/\Lambda \\
&+ \kappa_2 (\chi_1, \chi_2) \otimes (\chi_1, \chi_2) \otimes \chi_1^0 \otimes \Theta^n/\bar{\Lambda}^n + \kappa_3 \chi_{14} \otimes \chi_{14} \otimes \chi_1^0 \otimes \Theta^n/\bar{\Lambda}^n,
\end{aligned} \tag{114}$$

which is rewritten as

$$w_{\chi_1^0} = \kappa_1 [2\chi_1\chi_2\chi_3 + (\chi_1^2 - \chi_2^2)\chi_4] \chi_1^0/\Lambda + \kappa_2 \lambda^n (\chi_1^2 + \chi_2^2) \chi_1^0 + \kappa_3 \lambda^n \chi_{14}^2 \chi_1^0. \tag{115}$$

Then, we obtain

$$\kappa_1 [2\chi_1\chi_2\chi_3 + (\chi_1^2 - \chi_2^2)\chi_4] / \Lambda + \kappa_2 \lambda^n (\chi_1^2 + \chi_2^2) + \kappa_3 \lambda^n \chi_{14}^2 = 0. \tag{116}$$

Inserting $\chi_{14}^2 = -\frac{\eta_2}{\eta_3}(\chi_1^2 + \chi_2^2)$ in Eq. (71) into this equation, we have

$$\left[\left(\kappa_2 - \kappa_3 \frac{\eta_2}{\eta_3} \right) \lambda^n + \kappa_1 \chi_4 / \Lambda \right] \chi_1^2 + \left[\left(\kappa_2 - \kappa_3 \frac{\eta_2}{\eta_3} \right) \lambda^n - \kappa_1 \chi_4 / \Lambda \right] \chi_2^2 + 2\kappa_1 \chi_1 \chi_2 \chi_3 / \Lambda = 0. \tag{117}$$

Taking $\chi_3 \simeq 0$, we get

$$\frac{\chi_1^2}{\chi_2^2} = \frac{\kappa_1 \chi_4 - \left(\kappa_2 - \kappa_3 \frac{\eta_2}{\eta_3} \right) \lambda^n}{\kappa_1 \chi_4 + \left(\kappa_2 - \kappa_3 \frac{\eta_2}{\eta_3} \right) \lambda^n}. \tag{118}$$

As far as $\kappa_1 \chi_4$ is much larger than $(\kappa_2 - \kappa_3 \frac{\eta_2}{\eta_3}) \lambda^n$, the vacuum alignment $\chi_1 = \chi_2$ is guaranteed approximately. Therefore, magnitude of deviation of the vacuum alignment is parameter dependent. Calculating other next-to-leading operators, we can find easily that other vacuum alignment is deviated at most of order $\tilde{\alpha} \sim 0.01$.

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